Hokkaido Mathematical Journal Vol. 6 (1977) p. 39-45

Primitive extensions of rank 3 of $2^{n}\cdot GL(n, 2)$

By Shiro IWASAKI

(Received February 4, 1975)

1. Introduction.

As is well known, $(n+1)$-dimensional general linear group $GL(n+1,2)$ $=PSL(n+1,2)$ over $GF(2)$, the field with two elements is simple for $n\geqq 2$ and acts doubly transitively on $P(n, 2)$, the set of the points of n-dimensional projective space over $GF(2)$. Taking a point p in $P(n, 2)$, set $\Delta=P(n, 2)-\{p\}$ and let H be the stabilizer of p in $GL(n+1,2)$. Then H is the semi-direct product of an elementary abelian group of order 2^{n} and $GL(n, 2)$. The transitive permutation group (H, Δ) has rank 2 extension $(GL(n+1,2), P(n, 2))$. In this note, we determine primitive extensions of rank 3 of (H, Δ).

THEOREM. *Let* (G, Ω) *be a primitive extension of rank* 3 *of* (H, Δ). *Then*

(i) $n=1$ *and* (G, Ω) *is isomorphic to the dihedral group of order* 10 *acting on* 5 *letters, or*

(ii) $n=2$ *and* (G, Ω) *is isomorphic to the alternating group* A_{6} *acting on the unordered pairs of* $\{1,2,3,4,5,6\}$.

The idea of the proof of our Theorem is due to Bannai [2], which determined primitive extensions of rank 3 of $(PSL(n, 2^{f}), P(n-1,2^{f}))$, and the author thanks Dr. E. Bannai. He is also grateful to the referee for setting Lemma 7 a better form.

NOTATION. We follow the notation of Higman [4] mostly and use [4] frequently. In a transitive permutation group G on a finite set Ω, we denote by a^{g} the image of $a\in\Omega$ under $g\in G$, and for a subset X of Ω, G_{X} denotes the pointwise stabilizer of X, $G_{X}=\{g\in G|x^{g}=x$ for all $x\in X\}$. If $X=\{a, b, \cdots\}$, G_{X} is written $G_{ab\cdots}$. For a subset Y of G and $g\in G$, we let $Y^{g}=g^{-1}Yg$, $g^{Y}=\{g^{y}=y^{-1}gy|y\in Y\}$ and $a^{Y}=\{a^{y}|y\in Y\}$. The number of G_{a}-orbits $(a\in\Omega)$ counting $\{a\}$, is called the rank of (G, Ω).

The following notation will be fixed throughout this note. Let (G, Ω) be a primitive extension of rank 3 of (H, Δ), that is, 1) (G, Ω) is a primitive permutation group of rank 3, and 2) there exists an orbit $\Delta(a)$ of the stabilizer G_{a} of a point $a\in\Omega$ such that G_{a} acts faithfully on $\Delta(a)$ and $(G_{a}, \Delta(a))$ is isomorphic to (H, Δ).