Hokkaido Mathematical Journal Vol. 6 (1977) p. 102-129

Bochner-Minlos' theorem on infinite dimensional spaces

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(Received May 10, 1976)

§1. Introduction

In [2], Dao-Xing has shown that the following:

THEOREM A. *Let H and G be real separable Hilbert spaces such that H is a linear subspace of G and the inclusion mapping T from H into G is continuous. Let $\mathfrak{B}$ denote the totality of weak Borel sets in G, and $\mathfrak{F}$ the totality of weak Borel sets in the conjugate space H^* of H. Then, the following conditions are equivalent.*

(1) *T is a Hilbert-Schmidt operator from H into G.*

(2) *There exists a H-quasi-invariant finite measure (non-trivial) on $(G, \mathfrak{B})$.*

(3) *For any positive definite continuous function f on G with $f(0)=1$, there exists a unique probability measure μ on $(H^*, \mathfrak{F})$ such that, for any $x \in H$,*

$$f(x)=\int_{H^*} e^{ix^*(x)}\, d\mu(x^*).$$

In [20], the author has proven the following result. This is a generalization of Theorem A.

THEOREM B. *Let Φ be a separable σ-Hilbert space, with the inner products $(\varphi_1, \varphi_2)_n^{\Phi}$, and let Ψ be a linear subspace of Φ, and suppose that Ψ itself is a complete separable σ-Hilbert space with respect to the inner products $(\psi_1, \psi_2)_n^{\psi}$. Also, suppose that the inclusion mapping T from Ψ into Φ is continuous. For each n, let Φ_n denote the completion of Φ with respect to the inner products $(\varphi_1, \varphi_2)_n^{\Phi}$, and Ψ_n denote the completion of ψ with respect to the inner products $(\psi_1, \psi_2)_n^{\psi}$, respectively. Then, the following conditions are equivalent.*

(1) *T is a Hilbert-Schmidt operator from Ψ into Φ in σ-Hilbert spaces. Namely, for any m, there exists n such that T is a Hilbert-Schmidt operator from Ψ_n into Φ_m.*

(2) *For any n, there exists a Ψ-quasi-invariant finite measure (non-trivial) on $(\Phi_n, \mathfrak{B}_n)$.*

(3) *For any positive definite continuous function L on Φ with $L(0)=1$, there exists a unique probability measure μ on $(\Psi^*, \mathfrak{F})$ such that*