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Convolutions of measures on some thin sets

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1. Introduction.

Let G be a LCA group with dual $\hat{G}$, and E a compact subset of G. Following Rudin [4], we say that E is a Kronecker set if, to each $f\in C(E)$ with $|f|=1$ and $\varepsilon>0$, there exists $\gamma\in\hat{G}$ such that $\|f-\gamma\|_E<\varepsilon$. Let also $T=\{|z|=1\}$ be the circle group. Then in [2; Lemma 6.10], T. W. Körner proved that there exist two Kronecker sets K_1 and $K_2\subset T$, both homeomorphic to $D_2=\{0,1\}^\infty$, and two nonzero nonnegative Borel measures μ_1 on K_1 and μ_2 on K_2, such that $K_1+K_2=T$, and $\mu_1*\mu_2\in C^\infty(T)$.

In this paper, we prove analogs to the above result for general LCA groups. I thank Professor S. Saeki for his useful advices.

2. Notations.

Throughout this paper, G is used to denote a nondiscrete LCA group. We shall respectively denote by $A(G)=(L^1(\hat{G}))^\wedge$ and $C_c(G)$, the Fourier algebra of G, and the set of all continuous functions with compact support. Also we shall respectively denote by $M(G)$, $M_c(G)$, and $M_0(G)$, the measure algebra of all bounded regular measures on G, the set of all continuous measures in $M(G)$, and the set of those measures $\mu\in M(G)$ whose Fourier-Stieltjes transforms vanish at infinity. For $\mu\in M(G)$, $\|\mu\|$ will denote the total variation norm. The symbols $C_c^+(G)$, $M^+(G)$, and $M_c^+(G)$, will designate the set of all nonnegative functions in $C_c(G)$, the set of all nonnegative measures in $M(G)$, and the set of all nonnegative measures in $M_c(G)$, respectively. For μ, $\nu\in M(G)$, we write $\mu\perp\nu$ if μ and ν are mutually singular. We set $M_0^\perp(G)=\{\mu\in M(G)|\mu$ is singular with each $\nu\in M_0(G)\}$. For the other notation, we refer to Rudin [4].

DEFINITION 1. Let 0 be the unit of G. $q(G)=\sup\{s|$every neighborhood $0\in G$ contains an element of order $\geq s\}$.

DEFINITION 2. Let $q\geq 2$ be an integer, and E a totally disconnected compact subset of G. We say that E is a K_q-set if, to each $f\in C(E)$ with $f^q=1$, there exists $\gamma\in\hat{G}$ such that $f=\gamma$ on E.