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On the Hall-Higman and Shult theorems

By Tomoyuki YOSHIDA

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The purpose of this paper is to improve the proof of the wellknown Hall-Higman and Shult theorems. See also [3, Satz 5. 17. 13] and [5] for alternate proofs of these theorems.

THEOREM 1. ([4, Theorem 3. 1] and [2, Theorem B]). *Assume that the following hold*:

(a) *$G=PQ$ is a finite group, $Q=O_{p'}(G)$, $P=\langle x\rangle$ is a cyclic Sylow p-subgroup of G of order $p^n\neq 1$, and $O_p(G)=1$*;

(b) *k is a field of characteristic r and V is a faithful kG-module*;

(c) *V_P contains no kP-submodule isomorphic to kP, where V_P denotes the module V viewed as a kP-module.*

Let d be the degree of the minimal polynomial of x on V. Then there is a prime $q\neq p$ such that a Sylow q-subgroup of G is nonabelian and either

(1) *$q=2$, p is a Fermat prime, and $p^n-p^{n-1}\leq d<p^n$, or*

(2) *$p=2$, q is a Mersenne prime $<2^n$, and $2^nq/(q+1)\leq d<2^n$.*

We shall prove this theorem. By the induction argument ([1, Theorem 11. 1. 4] and [4, pp. 703-705]), the proof is reducted to the case where the following hypothesis is satisfied.

HYPOTHESIS 1. (1) The assumption of Theorem 1 holds;

(2) Q is an extraspecial q-group of order q^{2t+1}, q a prime, and P acts irreducibly on Q/Q' and trivially on Q';

(3) k is algebraic closed and Q acts irreducibly on V.

We shall prove the following theorem which is the main object in this paper. What it implies Theorem 1 is shown by an easy calculation.

THEOREM 2. *Assume that Hypothesis 1 holds. The V_P is isomorphic to the quotient module of a free kP-module by an irreducible submodule.*

To prove this theorem, we need two lemmas which are both wellknown.

LEMMA 3. *Let $G=PQ$ be a Frobenius group whose kernel Q is an elementary abelian minimal normal subgroup of G and complement P is*

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