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On integrability conditions on the space of sections of jet-bundles

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§ 1. Introduction

Let N be a n dimensional differentiable manifold. We consider a differentiable bundle $E(N)$ over N with projection π and the bundle $E^r(N)$ of r-jets of local sections of $E(N)$. Let Ω be an open set of $E^r(N)$. Then we let $\Gamma_\Omega E$ be the space of C^r sections, $s: N\rightarrow E(N)$ such that $j^r s(N)$ is contained in Ω equipped with C^r topology. We let $\Gamma_\Omega E^r$ be the space of continuous sections: $N\rightarrow E^r(N)$ whose image is contained in Ω, equipped with compact-open topology (An element of $\Gamma_\Omega E$ or $\Gamma_\Omega E^r$ will be called *Ω-regular*). Then there is a natural map $j^r: \Gamma_\Omega E\rightarrow\Gamma_\Omega E^r$. We discuss how the map j^r is close to a weak homotopy equivalence. This is related with the integrability of sections of $E^r(N)$ up to homotopy.

THEOREM. *Let Ω and Ω' be open sets in $E^r(N)$ with $\Omega\supseteqq\Omega'$. Let $\Omega-\Omega'$ is a finite union of regular submanifolds of Ω with codimensions greater than $n+\sigma$.*
(i) *If $j^r: \Gamma_{\Omega'}E\rightarrow\Gamma_{\Omega'}E^r$ is a τ-homotopy equivalence, then $j^r: \Gamma_\Omega E\rightarrow\Gamma_\Omega E^r$ is a* $\min(\tau, \sigma)$*-homotopy equivalence.*
(ii) *If $j^r: \Gamma_\Omega E\rightarrow\Gamma_\Omega E^r$ induces the isomorphisms of i dimensional homotopy groups $(0\leqq i\leqq\tau)$, then $j^r: \Gamma_{\Omega'}E\rightarrow\Gamma_{\Omega'}E^r$ induces the isomorphisms of i dimensional homotopy groups* $(0\leqq\tau\leqq\min(\tau, \sigma)-1))$.

A j-homotopy equivalence means the isomorphisms of i dimensional homotopy groups $(0\leqq i<j)$ and a surjection of j dimensional homotopy groups.

This theorem is a generalization of Transversality lemma due to A. du Plessis in [5] which is the case of differentiable maps of the above theorem. The applications of the theorem are given in § 4 to the cace of Thom-Boardman singularities ([2, 7, 9]). The proof is based on the transversality arguments.

All manifolds should be paracompact and Hausdorff.

§ 2. A variant of Thom's transversality theorem

In this section we will show a variant of Thom's transversality theorem.