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On the equilibrium existence in abstract economies

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§ 1. Introduction

The purpose of the present paper is to extend earlier equilibrium existence theorems for economies with finitely many agents and with finite-dimensional commodity spaces (see [1] and [8]) to the case of infinite-dimensional commodity spaces. The utilization of Δ or Δ_0, whose existence in the infinite-dimensional space was shown in [12], as the set of price systems is indispensable when it is intended to guarantee the existence of equilibrium for generalized economies. The extension also concerns the recent results to eliminate unnecessary assumptions on consumers' preferences for the proof of equilibrium existence ([4], [6], [9], [10], [11]).

After summarizing the useful auxiliary theorems concerning semi-continuous set-valued mappings in § 2, three types of economies are dealt with in the last three sections of the present paper, respectively. Individual preferences are given in the following three ways: (1) by the utility functions, (2) by the binary relations and (3) by the preference mappings. The present analysis is limited to the pure exchange model only for the sake of conciseness and clarity.

§ 2. Auxiliary theorem concerning semi-continuous set-valued mappings.

Let F be a set-valued mapping assigning to each $x\in X_1$ a subset $F(x)$ of X_2 where X_1 and X_2 are topological spaces. F is called *lower semi-continuous* (briefly l. s. c.) *at* $x_0\in X_1$, if for each open set G meeting $F(x_0)$ there exists a neighbourhood $V(x_0)$ of x_0 such that $F(x)\cap G\neq\phi$ for all $x\in V(x_0)$. F is called *upper semi-continuous* (briefly u. s. c.) *at* $x_0\in X_1$, if for each open set $G\supset F(x_0)$ there exists a neighbourhood $V(x_0)$ of x_0 such that $F(x)\subset G$ for all $x\in V(x_0)$. F is called *lower semi-continuous in* X_1 (briefiy l. s. c. in X_1), if it is lower semi-continuous at each point of X_1. F is a called *upper semi-continuous in* X_1 (briefly u. s. c. in X_1), if it is upper semi-continuous at each point of X_1 and the set $F(x)$ is compact for each $x\in X_1$. If F is both l. s. c. in X_1 and u. s. c. in X_1, then it is called *continuous in* X_1. F