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On the normal p-structure of afinite group and related topics I

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To Reinhold Baer, on his seventy-fifth birthday, 22 July 1977

1. Introduction.

In this note we describe (for any prime p) group theoretic properties p^* and p^*p and the corresponding functors O_{p^*} and O_{p^*p}.

They are in the class of all (finite!) groups what the functors $O_{p'}$ and $O_{p',p}$ are for solvable or, more precisely, for p-constrained groups.

Besides this analogy $\begin{matrix} p^*\leftrightarrow p' \\ p^*p\leftrightarrow p',p \end{matrix}$ there is another interesting analogy, namely between O_{p^*}, O_{p^*p} and the well-known functors E and F^*, respectively. In order to exhibit this analogy most clearly we treat these four functors (and the corresponding properties) uniformly (in section 4).

This is done via the concept of the generalized centralizer $C_G^*(X)$ discussed in section 3, and by working with a prime set π which is the set of all primes or consists of our p only.

In sections 5 and 6 we specialize to these two cases, getting the (well known) elementary $E-F^*$-theory and our p^*-p^*p-theory.

In the theory of simple groups (general classification problems) one has reached a point where one is forced to handle nearly arbitrary (sub) groups H, and hence needs small subgroups of H conveniently structured which still control the structure of H somehow. It is exactly this what $E(H)$, $F^*(H)$, $O_{p^*}(H)$, $O_{p^*p}(H)$ and similar constructions are all about. In this field however, due to the structure of the known simple groups, $O_{p^*}(H)$ and $O_{p^*p}(H)$ appear in a certain special form, and then many of our results are contained in the work of Gorenstein and Walter, see [2], 17 2, [3], [4], [5]. These include our Theorem 6. 10 stating that $O_{p^*}(N_H(P))\subseteq O_{p^*}(H)$ for every p-subgroup P.

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