Hokkaido Mathematical Journal Vol. 7 (1978) p. 252–258

The Bochner curvature tensor on almost Hermitian manifolds

By L. VANHECKE and D. JANSSENS

(Received June 22, 1977; Revised September 26, 1977)

Abstract. We prove a decomposition theorem for curvature tensors on a Hermitian vector space over $\boldsymbol{R}$ and use this to introduce Bochner curvature tensors. Applications which include the well known Kähler case are given for almost Hermitian manifolds.

0. Introduction

Singer and Thorpe [11] established a natural decomposition of curvature tensors on an n-dimensional real vector space with inner product and Nomizu [10] used this decomposition to study generalized curvature tensor fields. Kowalski [8] considered also a decomposition theory to study conformal differential geometry. In these papers the Weyl conformal curvature tensor is obtained in a very natural way as a projection of the Riemann curvature tensor.

Sitaramayya [12] and Mori [9] gave a similar decomposition to study curvature tensors on Kähler manifolds.

In this paper we extend these results, based on [14]. First we prove a decomposition theorem for a class of curvature tensors L on a Hermitian vector space V and derive the Bochner curvature tensor associated with L. Then we consider a large class of almost Hermitian manifolds and study some properties of the Bochner curvature tensor field associated with the Riemann curvature structure.

1. Curvature tensors

Let V be an n-dimensional real vector space with inner product g. A tensor L of type (1, 3) over V is a bilinear mapping $L: V\times V\to \mathrm{Hom}(V, V)$: $(x, y)\to L(x, y)$. L is called a *curvature tensor* on V if it has the following properties:

AMS (MOS) Subject classifications (1970). Primary 53B35, 53C55. Secondary 53A45.
Key words and phrases. Curvature tensors, decomposition theory, Bochner tensor, almost Hermitian manifolds.