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Characteristic mixed problems for hermitian systems in three unknowns

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§ 1. Introduction and results

The purpose of this paper is to prove that the results of Strang [10] for 2×2 systems are also valid for hermitian 3×3 systems with characteristic boundary including the linearized shallow water equations. This has been conjectured by Majda and Osher [5].

We consider the mixed problems for hermitian systems of first order in the quarter space $t\geqq 0$, $x\geqq 0$, $y=(y_1, \cdots, y_n)\in \boldsymbol{R}^n$:

$$
(1.1)\qquad
\begin{aligned}
&\frac{\partial u}{\partial t}+A\frac{\partial u}{\partial x}+\sum_{j=1}^{n}A_j\frac{\partial u}{\partial y_j}=f && \text{in } t>0,\ x>0,\ y\in \boldsymbol{R}^n,\\
&u(0, x, y)=u_0(x, y) && \text{in } x>0,\ y\in \boldsymbol{R}^n,\\
&Bu(t, 0, y)=0 && \text{in } t>0,\ y\in \boldsymbol{R}^n.
\end{aligned}
$$

Here we assume A and A_j to be constant, hermitian 3×3 matrices, the boundary $x=0$ to be characteristic; that is, $\det A=0$. Furthermore, we assume B to be a constant $l\times 3$ matrix whose rank l is equal to the number of positive eigenvalues of A. In the treatment of characteristic mixed problems it is natural to assume that the problem (1. 1) is reflexive, that is, $\ker A\subset \ker B$ (see Kubota and Ohkubo [4] and Rauch [9]).

Our problem is whether there exists a solution u of (1. 1) satisfying the following energy inequality: There is a constant $C_T>0$ for each $T>0$ such that

$$
(1.2)\qquad \|u(t)\|\leqq C_T\left(\|u_0\|+\int_0^t\|f(s)\|\,ds\right)
$$

for any t with $0\leqq t\leqq T$. Here $\|\cdot\|$ stands for the usual L^2-norm in the half space $x>0$, $y\in \boldsymbol{R}^n$.

A sufficient condition for the existence of a solution of (1. 1) satisfying (1. 2) has been already established by Friedrichs [2] and Lax and Phillips [7]. This condition is called "maximally non-positive"; that is, after a non-singular transformation $v=Tu$ of unknowns such that $A'=T^{-1}AT$ and $A'_j=T^{-1}A_jT$ are hermitian, it holds that