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Numerical ranges of the tensor products of elements

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Abstract. We will discuss the convexoid, normaloid and spectraloid elements in a unital Banach algebra A, and in the tensor product $A_1 \otimes_\alpha A_2$ of two unital Banach algebras A_1, A_2 under some compatible reasonable norm α. If A is a Hilbert space, the convexoid, normaloid and spectraloid operators on A are investigated by Halmos, Furuta, Nakamoto, Takeda and Saito etc. Moreover, we give necessary and sufficient conditions for the joint convexoidity of n-tuple of operators on Hilbert spaces.

1. Introduction.

Recently, Bonsall and Duncan in [1] developed the numerical range $V(T)$ for general normed linear space A which is defined by

$$V(T)=\left\{f(Tx):(x,f)\in\pi\right\}$$

where $\pi=\{(x,f)\in S(A)\times S(A^*):f(x)=1\}$, and $S(A)$ denotes the unit sphere of A and A^* is the dual space of A.

Let A be a normed algebra with unit 1. For $a\in A$, the numerical range $V(a)$ of the element a is defined by

$$V(a)=V(T_a),$$

where T_a is the left regular representation (operator) on A. It is remarkable that $V(a)$ can be expressed by $V(a)=\{f(a):f\in D(A,1)\}$, where $D(A,1)=\{f\in A^*:\|f\|=1=f(1)\}$ (cf. Bonsall and Duncan [1]). The numerical range $W(T)$ of the operator T on Hilbert space (cf. Halmos [6]) is convex, but in general $V(T)$ is not convex. While $V(a)$ is known to be a compact convex set (cf. Bonsall and Duncan [1]). In this note we discuss the numerical range of a Banach algebra with unit, and consider $a\in A$ such that the numerical range of the element a coincides with the convex hull of its spectrum; for such element we shall say that a is convexoid. It seems not to be known whether the tensor product of convexoid elements x, y

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