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Notes on homogeneous almost Hermitian manifolds

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1. Introduction

The object of this paper is to characterize homogeneous almost Hermitian manifolds (i. e., almost Hermitian manifolds with transitive automorphism groups) in terms of the behavior of their curvature tensors and structure tensors under parallel translations. In [1], W. Ambrose and I. M. Singer have characterized homogeneous Riemannian manifolds in terms of the behavior of their curvature tensors under the parallel translations. They have proved the following

THEOREM A. *Let* $(M, \langle\ ,\ \rangle)$ *be a homogeneous Riemannian manifold. Then, there exists a skew-symmetric tensor field* T *of type* $(1, 2)$ *on* M *satisfying*

(A) $\nabla_X R = T(X)\cdot R$,

(B) $\nabla_X T = T(X)\cdot T$, *for any tangent vector* $X \in T_x(M)$, $x \in M$.

Conversely, if a connected, simply connected complete Riemannian manifold $(M, \langle\ ,\ \rangle)$ *admits a skew-symmetric tensor field* T *of type* $(1, 2)$ *on* M *satisfying* (*A*) *and* (*B*), *then* $(M, \langle\ ,\ \rangle)$ *is a homogeneous Riemannian manifold.*

In this note, we shall prove the following

THEOREM B. *Let* $(M, J, \langle\ ,\ \rangle)$ *be a homogeneous almost Hermitian manifold. Then, there exists a skew-symmetric tensor field* T *of type* $(1, 2)$ *satisfying* (*A*), (*B*) *in Theorem A, and furthermore*

(C) $\nabla_X J = T(X)\cdot J$, *for any tangent vector* X.

Conversely, if a connected, simply connected, complete almost Hermitian manifold $(M, J, \langle\ ,\ \rangle)$ *admits a skew-symmetric tensor field* T *of type* $(2, 1)$ *satisfying* (*A*), (*B*) *and* (*C*), *then* $(M, J, \langle\ ,\ \rangle)$ *is a homogeneous almost Hermitian manifold.*

By slight modification of the proof of Theorem A, we may prove Theorem B (cf. § 3, § 4). As a result of Theorem B, we have the following well-known result. If $(M, J, \langle\ ,\ \rangle)$ is a connected, simply connected Hermitian symmetric space, then it is homogeneous almost Hermitian manifold.