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Some considerations on fibred spaces with certain almost complex structures

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Fibred spaces with almost complex structures have been studied by M. Ako [1][1)] and B. Watson [2]. The interesting result on a fibred space with Kählerian structure was given in [1]. The purpose of the present paper is to study the analogous problem in fibred almost Kählerian and almost Tachibana spaces and give certain extensions of Theorem 5.1 in [1]. For the purpose we need to have the method in [1].

In section 1 we define fibred spaces $\tilde{M}$ and the additional conception. In section 2 we introduce a projectable Riemannian metric $\tilde{g}$ in $\tilde{M}$. In section 3 we give formulas for the covariant differentiation with respect to the Riemannian connection induced by $\tilde{g}$. In section 4 we give some lemmas which will be used to prove Theorems in section 5.

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1. Fibred spaces.

The manifolds, objects and mappings which we consider are assumed to be of class C^{∞}. The notation used in this paper is the same as [1].

Let $\tilde{M}$ and M be manifolds of dimension n and m respectively, where $n>m$. A mapping σ from $\tilde{M}$ onto M is called a submersion if the differential map σ_* induced by σ has the maximal rank m everywhere in $\tilde{M}$. We assume the existence of such a submersion. $(\tilde{M}, M; \sigma)$ is then called a fibred space over M. Under the above assumption the inverse image $\mathscr{F}_P$ of $P\in M$ by σ is an $(n-m)$-dimensional closed submanifold of $\tilde{M}$ and is called a fibre over P. Throughout this paper we assume that each fibre is connected.

A vector in $\tilde{M}$ at $\tilde{P}\in\tilde{M}$ is said to be vertical if it is tangent to the fibre over $\sigma(\tilde{P})$. A vector field of vertical vectors is called a vertical vector field.

Now, since the rank of $\sigma_*=m$, there are $(n-m)$ linearly independent vertical vector fields $C_\alpha(\alpha=m+1, \cdots, n)$ in a neighborhood of each point in

1) Numbers in brackets refer to references at the end of the paper.