Hokkaido Mathematical Journal Vol. 9 (1980) p. 91-106

Certain free cyclic group actions on homotopy spheres, bounding parallelizable manifolds

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(Received March 24, 1979)

Introduction

The purpose of this paper is to study free cyclic group actions on homotopy spheres constructed by S. Weintraub [3]. He applied an equivariant plumbing technique to construct semifree cyclic group actions on highly-connected $4k$-dimensional manifolds. The boundaries of these manifolds are elements of bP_{4k} and they admit free Z_p-actions for any integer p. We call these "Weintraub's actions".

On the other hand, it is well known that López De Medrano [1] constructed free involutions on homotopy spheres with non-trivial Browder-Liversay invariants. It is apparent that López's construction cannot be applied to get any other free cyclic group actions except involutions. However, we shall prove that certain examples of López's involutions on homotopy spheres of bP_{4k} extend to free Z_{2q}-actions for any q which are realized by "Weintraub's actions" raised above. This is the main motivation of this research.

The results are summarized as follows. One of the properties about Weintraub's actions has been found in [3, Theorem 1.7] and in § 2, we state this in an alternative form for our argument.

THEOREM 1. *Suppose that p is any integer. Choose a unimodular, even, symmetric matrix A and denote $\sigma(A)$ its index. For any $k\geqq 2$ and collection $\{a_1, \cdots, a_k\}$ with $(a_i, p)=1$, there is a free Z_p-action T_A on a homotopy sphere $\Sigma_A\in bP_{4k}$ the Atiyah-Singer invariant of which has the form*

$$\sigma(T_A, \Sigma_A^{4k-1})=\prod_{i=1}^{k}\left(\frac{1+t^{a_i}}{1-t^{a_i}}\right)^2-\sigma(A)\,.$$

Here $\Sigma_A=\sigma(A)/8\Sigma_1$, where Σ_1 is the generator of bP_{4k} and $t=\exp(2\pi i/p)$.

Hereafter, by (T_A, Σ_A) we denote the free Z_p-action on the homotopy sphere constructed for any p and A under the assumptions of Theorem 1. As to the normal cobordism classes of such actions, we shall prove the following result.