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# Conformally flat Riemannian manifolds of constant scalar curvature

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## Introduction

N. Ejiri [3] showed the existence of compact Riemannian manifolds of constant scalar curvature which admit non-homothetic conformal transformations. This is related to solutions of a non-linear differential equation (*) and he did not give any concrete solutions.

Here we give explicit solutions of (*) for the case of $n=3$ (Lemma 4). This problem is also related to examples of compact or complete conformally flat Riemannian manifolds of constant scalar curvature $S$. In § 2 we show concrete examples of such Riemannian manifolds (Theorems 6 and 7). These show that $S=$ constant (as one condition of weaker type of local homogeneity) on a conformally flat Riemannian manifold does not imply local homogeneity.

A Kählerian analogue of conformal flatness is the vanishing of the Bochner curvature tensor. In § 3 we study some conditions weaker than local homogeneity. Contrary to the conformally flat case, Theorems 8 and 11 show that $S=$ constant or constancy of length of the Ricci curvature tensor on a Kählerian manifold with vanishing Bochner curvature tensor implies local homogeneity.

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## § 1. Warped products.

Let $(F, h)$ be an $n$-dimensional Riemannian manifold and $f$ be a positive function on an open interval $I$ of a real line $\boldsymbol{R}$. Consider the product $I\times F$ with the projections $\pi: I\times F\rightarrow I$, and $\eta: I\times F\rightarrow F$. The space $I\times F$ with the Riemannian metric

$$\langle X, Y\rangle_{(t,x)}=(\pi X, \pi Y)_{t}+f^{2}(\pi x)\, h_{x}(\eta X, \eta Y)$$

is called the warped product and is denoted by $I\times_{f}F$, where $X$, $Y$ are tangent vectors at $(t, x)\in I\times F$, and $\pi$, $\eta$ denote also their differentials (cf. R. L.