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# Remarks on the spaces of type $H+AP$

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## § 1. Introduction.

For a LCA group $G$, $AP(G)$ and $M(G)$ denote the space of all almost periodic functions and the space of all bounded regular measures on $G$ respectively.

Let $R$ be the reals. In [4], S. Power proved that the sum of the Hardy space and the space of all almost periodic functions on $R(H^{\infty}(R)+AP(R))$ is a closed subspace of $L^{\infty}(R)$ but not an algebra.

For a LCA group $G$, in [6], W. Rudin proved that $H+C_{u}(G)$ is a closed subspace of $L^{\infty}(G)$ for a translation-invariant weak*-closed subspace $H$ of $L^{\infty}(G)$ and the space of all bounded uniformly continuous functions $C_{u}(G)$.

In this paper, we shall prove that $H+AP(G)$ is a closed subspace of $L^{\infty}(G)$ for every translation-invariant weak*-closed subspace $H$ of $L^{\infty}(G)$. Moreover, we shall investigate whether a space of type $H+AP(G)$ becomes an algebra.

DEFINITION 1. *For any subset $\Phi$ of $L^{\infty}(G)$, the spectrum of $\Phi$ is defined as the set $\sigma(\Phi)$ of all $\gamma\in\hat{G}$ that belong to the smallest translation-invariant weak*-closed subspace of $L^{\infty}(G)$ containing $\Phi$.*

Easily, we have the following:

$$\sigma(\Phi)=\cap\left\{\hat{f}^{-1}(0)\,;\,f\in L^{1}(G),\ f_{*}\Phi=0\right\}.$$

## § 2. Main Theorem

Let $\bar{G}$ denote the Bohr compactification of $G$. Then we can identify $AP(G)$ with $C(\bar{G})$. Let $d\bar{x}$ denote the Haar measure on $\bar{G}$. For $f$, $g\in AP(G)$, we define $f*g$, $||f||_{1}$ and $\hat{f}$ with respect to $d\bar{x}$. The symbol $B(L^{\infty}(G))$ denotes the Banach algebra of bounded linear operators on $L^{\infty}(G)$.

LEMMA. *There exists a linear map*

$$f\longmapsto\lambda_{f};\ AP(G)\longmapsto B\left(L^{\infty}(G)\right)$$

*satisfying the following conditions for $f$, $g\in AP(G)$ and $\phi\in L^{\infty}(G)$:*