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Polynomial rings over Krull orders in simple Artinian rings

Dedicated to professor Goro Azumaya
for his 60th birthday

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Introduction

Let Q be a simple artinian ring. An order R in Q is called Krull if there are a family $\{R_i\}_{i\in I}$ and $S(R)$ of overrings of R satisfying the following:

(K 1) $R=\cap_{i\in I}R_i\cap S(R)$, where R_i and $S(R)$ are essential overrings of R (cf. Section 2 for the definition), and $S(R)$ is the Asano overrig of R;

(K 2) each R_i is a noetherian, local, Asano order in Q, and $S(R)$ is a noetherian, simple ring;

(K 3) if c is any regular element of R, then $cR_i\neq R_i$ for only finitely many i in I and $R_kc\neq R_k$ for only finitely many k in I.

If $S(R)=Q$, then R is said to be bounded. Author mainly investigated the ideal theory in bounded Krull orders in Q (cf. [10], [11], [12] and [13]). The class of Krull orders contains commutative Krull domains, maximal orders over Krull domains, noetherian Asano orders and bounded noetherian maximal orders. It is well known that if D is a commutative Krull domain, then the polynomial and formal power series rings $D[\boldsymbol{x}]$ and $D[[\boldsymbol{x}]]$ are both Krull, where the set $\boldsymbol{x}$ of indeterminates is finite or not.

The purpose of this paper is to show how the results above can be carried over to non commutative Krull orders by using prime v-ideals and localization functors. After giving some fundamental properties on polynomial rings (Section 1), we shall show, in Section 2, that if R is a Krull order in Q and if $\boldsymbol{x}$ is a finite set, then so is $R[\boldsymbol{x}]$. In case $\boldsymbol{x}$ is an infinite set, we can not show whether $R[\boldsymbol{x}]$ is Krull or not. But we shall show that $R[\boldsymbol{x}]$ satisfies some properties interesting in multiplicative ideal theory as follows:

(i) $R[\boldsymbol{x}]=\cap_P R[\boldsymbol{x}]_P\cap S(R[\boldsymbol{x}])$, where P ranges over all prime v-ideals of $R[\boldsymbol{x}]$, the local ring $R[\boldsymbol{x}]_P$ is a noetherian and Asano order in the quotient ring of $R[\boldsymbol{x}]$ and the Asano overring $S(R[\boldsymbol{x}])$ is a simple ring.

(ii) The integral v-ideals of $R[\boldsymbol{x}]$ satisfies the maximum condition.