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Q-projective transformations of an almost quaternion manifold: II

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Continued to the previous paper ([8]), we shall study infinitesimal Q-projective transformations on the quaternion Kählerian manifold[3] and prove the following theorems:

THEOREM 4. *If a complete quaternion Kählerian manifold* (M, g, V) *with positive scalar curvature* S *admits an infinitesimal non-affine* Q-*projective transformation,* (M, g, V) *is isometric to the quaternion projective space of constant* Q-*sectional curvature* $S/4m(m+2)$.

THEOREM 5. *In a compact quaternion Kählerian manifold, each vector field which satisfies* (3.4) *is an infinitesimal* Q-*projective transformation.*

Concerning infinitesimal projective transformations of a Riemannian manifold or infinitesimal holomorphically projective transformations of a Kählerian manifold, we have known interesting analogous results, and we can see them in [9], [10], [11], and etc..

§ 5. Proof of Theorem 4.

From (3.4), $\cdots$, (3.7) and Ricci's formula, we get

$$(5.1)\qquad \begin{aligned} 4(m+1)\nabla_j\eta_i &= \nabla_j(\nabla_i\nabla_h X^h - \nabla_h\nabla_i X^h) + \nabla_j\nabla_h\nabla_i X^h \\ &\quad - \nabla_h\nabla_j\nabla_i X^h + \nabla_h\nabla_j\nabla_i X^h \\ &= -S(\nabla_j X_i + \nabla_i X_j)/4m + 2\nabla_j\eta_i - 2\Lambda_{ji}^{kh}\nabla_k\eta_h\,. \end{aligned}$$

Transvecting (5.1) by Λ_{fg}^{ji} and substituting it into (5.1), we have

$$(5.2)\qquad \nabla_j\eta_i = S\Big\{\Lambda_{ji}^{kh}(\nabla_k X_h + \nabla_h X_k) - (2m+3)(\nabla_j X_i + \nabla_i X_j)\Big\}\Big/32m^2(m+2)$$

where indices f and g run over the range $\{1, \cdots, 4m\}$. On the other hand, from (1.1) and (3.1), we have

3) We assume that the dimension $4m$ of $M \geqq 8$.