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On a class of pseudo-differential operators and hypoellipticity

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0. Introduction

In this paper we shall consider a class of pseudo-differential operators P whose characteristic set Σ is the union of closed conic submanifolds Σ_1, $\Sigma_2, \cdots, \Sigma_n$. Under some transversarity conditions and involutiveness, we shall give the necessary and sufficient condition for hypoellipticity of P.

When $n=1$, our class coincides with $L^{m,M}_{k}(X, \Sigma)$ introduced by Helffer [5] and moreover if $k=2$, it coincides with $L^{m,M}_{c}(X, \Sigma)$ introduced by Sjöstrand [8] (see also [4]). In the case where $n=1$, $M=2$, $k=2$ and Σ is involutive, Boutet de Monvel [1] gives a necessary and sufficient condition for the existence of a parametrix of P in $OPS^{-m,-M}$ (more general class than ours OPL), which is also equivalent to the hypoellipticity of P with loss of 1-derivative. For general M and k, [5] constructs a left parametrix and then proves hypoellipticity with loss of M/k-derivatives, which is a generalization of [1].

In § 1, using the technique developed by [5], we introduce an invariance of P (Theorem 1.3) and state a necessary and sufficient condition for the hypoellipticity of P (Theorem 1.5). In § 2 and § 3, we give their proofs. § 4 is devoted to the study of hypoellipticity for another class of pseudo-differential operators on $\boldsymbol{R}^N$.

1. Notations, Definitions and Statements of the results

Let X be a paracompact C^∞ manifold of dimension N and let $T^*(X)-\{0\}$ be the cotangent bundle minus the zero section.

DEFINITION 1.1. *Let* $\Sigma_1, \Sigma_2, \cdots, \Sigma_n$ *be closed conic submanifolds of codimension* $p_1, p_2, \cdots, p_n$ *respectively in* $T^*(X)-\{0\}$ *and let* $m\in\boldsymbol{R}$, $M_1, M_2, \cdots, M_n\in\boldsymbol{Z}^+$, $k_1, k_2, \cdots, k_n\in\boldsymbol{Z}^+$ *and* $k_j\geqq 2$, $j=1,2,\cdots,n$. *Then we define* $OPL^{m,M_1,M_2,\cdots,M_n}_{k_1,k_2,\cdots,k_n}(X; \Sigma_1, \Sigma_2, \cdots, \Sigma_n)$ *to be the space of pseudo-differential operators* P *which, in every local coodinate system* $U\subset X$, *has a symbol of the form*