Hokkaido Mathematical Journal Vol. 9 (1980) p. 36-45

Stability of G-unfoldings

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(Received December 26, 1978)

§ 0. Introduction

In [4], R. Thom has presented the problem to study the bifurcation of singularities of G-invariant functions. (Where G is a compact Lie group). In realtion to this problem, G. Wassermann has classified singularities with compact abelian symmetry and their universal G-unfoldings ([6]). But, from the view point of "Catastrophe theory" we must classify stable G-unfoldings instead of universal G-unfoldings.

In this paper, we will prove the equivalence of these notions of G-unfoldings. Once this is proved, the list of universal G-unfoldings in [6] can be exchanged for stable G-unfoldings.

The main result of this paper will be formulated in § 1. Preliminary facts about G-invariant functions and jet bundles are contained in § 2. Proof of the main result will be given in § 3.

All functions and actions of Lie group should be smooth.

§ 1. Formulation of the result

Let G be a compact Lie group which acts linearly on $\boldsymbol{R}^n$. We shall denote $C^\infty(\boldsymbol{R}^n)$ the set of all C^∞-functions over $\boldsymbol{R}^n$; $C_0^\infty(\boldsymbol{R}^n)$ the set of all C^∞-function germs at 0. We shall set $\mathfrak{M}_0^\infty(\boldsymbol{R}^n):=\{f\in C_0^\infty(\boldsymbol{R}^n)|f(0)=0\}$. Then $C_0^\infty(\boldsymbol{R}^n)$ is an $\boldsymbol{R}$-algebra in the usual way, and $\mathfrak{M}_0^\infty(\boldsymbol{R}^n)$ is its unique maximal ideal.

A function $f\in C^\infty(\boldsymbol{R}^n)$ will be said to be *G-invariant* if $f(gx)=f(x)$ for any $x\in\boldsymbol{R}^n$ and $g\in G$. The set of G-invariant functions over $\boldsymbol{R}^n$ will be denoted by $C^G(\boldsymbol{R}^n)$ and the set of all G-invariant function germs at 0 denoted by $C_0^G(\boldsymbol{R}^n)$; it is a subalgebra of $C_0^\infty(\boldsymbol{R}^n)$, and $\mathfrak{M}_0^G(\boldsymbol{R}^n):=C_0^G(\boldsymbol{R}^n)\cap\mathfrak{M}_0^\infty(\boldsymbol{R}^n)$ is its unique maximal ideal.

Let $f:(\boldsymbol{R}^n, a)\longrightarrow(\boldsymbol{R}, c)$ and $h:(\boldsymbol{R}^n, a')\longrightarrow(\boldsymbol{R}, c')$ be germs of G-invariant functions at a and a' $(f(a)=c, f(a')=c')$. We shall say f is *G-right equivalent* to h (and we shall write $f\sim_G h$) if there is a equivariant diffeomorphism germ $\phi:(\boldsymbol{R}^n, a)\longrightarrow(\boldsymbol{R}^n, a')$ such that $f=h\circ\phi+(c-c')$.

DEFNITION 1.1. Let $f\in\mathfrak{M}_0^G(\boldsymbol{R}^n)$. We say f is *strongly k-determined*