Hokkaido Mathematical Journal Vol. 9 (1980) p. 1-17

On a parametrix for the hyperbolic mixed problem with diffractive lateral boundary II

By Taira SHIROTA

(Received July 11, 1979)

§ 1. Introduction.

In the previous paper ([5]) we proved that there exist parametrices with a loss of 1/3 derivative in a neighborhood of a fixed diffractive point for second order normally hyperbolic mixed problems with boundary conditions more general than Dirichlet's or Neumann's.

The aim of the present paper is to give examples of the mixed problems with such general boundary conditions mentioned above, which may appear in classical Mathematical Physics in a natural way. Namely, we consider the equation with given initial data at $t=0$

$$\partial_t^2 u=\mu\Delta u+(\lambda+\mu)\nabla(\nabla\cdot u) \qquad \text{for } t\geqq 0 \tag{1.1}$$

where $u={}^t(u_1, u_2)$ is the displacement vector and λ, μ the Lamé parameters of the medium which occupies a C^∞-domain Ω of R^2. Denote the stress-strain components by

$$\sigma_{ij}=\lambda(\nabla\cdot u)\,\delta_{ij}+\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right), \qquad i,j=1,2$$

and let

$$\begin{pmatrix} a(X), & -b(X) \\ b(X), & a(X) \end{pmatrix} \in O(2)$$

which is a C^∞-cross-section over $\partial\Omega$. Now we impose the following mixed boundary conditions :

$$\begin{cases} a(X)\,u_1(X)-b(X)\,u_2(X)=0\,, \\ \sum_{k=1,2}\bigl(b(X)\,\sigma_{1k}(X)+a(X)\,\sigma_{2k}(X)\bigr)\,n_k(X)=0 \quad \text{on} \quad \partial\Omega\,. \end{cases} \tag{1.2}$$

Here $(n_1, n_2)(X)$ is the unit inward normal at $X=(x_1, x_2)\in\partial\Omega$ (see [14]). Our boundary conditions mean that for any boundary point X they are rigid in the direction $(a(X),\ -b(X))$ and are free for the direction $(b(X),\ a(X))$.

To make (1.1) a normally hyperbolic system we must assume that