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On the first eigenvalue of the Laplacian acting on p-forms

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§ 1. Introduction.

Let M be a compact and oriented Riemannian manifold isometrically immersed in a complete and simply connected space form of constant sectional curvature K. Let $\lambda_1^p(M)$ denote the first non-zero eignvalue of the Laplacian acting on p-forms on M. In this note we will be concerned with the next problem :

Estimate $\lambda_1^p(M)$ from above in terms of the quantities determined by M (the volume $Vol(M)$, the diameter $d(M)$, etc.) and the immersion (the mean curvature vector field η, the second fundamental form S, etc.). Furthermore, give the equality condition.

For this problem, in the case $K=0$, Bleecker-Weiner ([1]) obtained an estimate of $\lambda_1^0(M)$. Masal'cev ([4]) also obtained the same result by a different method but under the additional assumption that M is a hypersurface. Masal'cev ([5]) gave an estimate of $\lambda_1^p(M)$ for $1\leqslant p\leqslant \dim M-1$, when M is a hypersurface. In the case $K\neq 0$, Masal'cev ([6]) obtained an estimate of $\lambda_1^0(M)$ for a hypersurface without the equality condition. Now we can compare the first eigenvalue $\lambda_1^p(M)$ with that of the standard m-sphere $S^m(1)$ of constant curvature 1. The purpose of the present note is to present the following

THEOREM. *Let M be an $m(\geqslant 2)$-dimensional compact and oriented Riemannian manifold without boundary.*

(A) *If M is isometrically immersed in the n-dimensional Euclidean space E^n, then for $0\leqslant p\leqslant m$, we have*

$$\lambda_1^p(M)\leqslant \frac{\lambda_1^p(S^m(1))}{m\,Vol(M)}\int_M ||S||^2 dV_M\,. \tag{1.1}$$

Equality holds iff M is embedded as a geodesic sphere in some $(m+1)$-dimensional totally geodesic submanifold in E^n. Here $||S||$ denotes the length of S and dV_M the volume form of M.

(B) *If M is isometrically immersed in $S^n(1)$, then for $1\leqslant p\leqslant m-1$, we have*