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Boundary behavior of Dirichlet solutions at regular boundary points

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Let G be a bounded domain in the complex plane. Let f be an extended real-valued continuous function on the boundary ∂G of G. If f is bounded, there exists the Dirichlet solution H_f^G ([2]) and if p_0 is a regular boundary point, then

$$\lim_{G\ni z\to p_0} H_f^G(z)=f(p_0)\,. \tag{1}$$

From M. Brelot's example ([1]) we can make an example which violates $\lim_{G\ni z\to p_0} H_f^G(z)=f(p_0)$ for an unbounded continuous resolutive f at a regular boundary point p_0. Here we show that under a certain condition (1) holds for an unbounded continuous resolutive f at a regular boundary point p_0. Set $f^+=\max\{f, 0\}$ and $f^-=\max\{-f, 0\}$. Our result is the following.

THEOREM. *Let G be a bounded domain in the complex plane and p_0 be a regular boundary point of G. Let f be an extended real-valued continuous and resolutive boundary function on ∂G.*

(i) *The case where $f(p_0)$ is finite.* $\lim_{G\leftarrow\ni pz} H_f^G(z)=f(p_0)$ *holds if* $\iint_{G\cap V} H_{|f|}^G(z)\,dxdy<\infty$ *for a neighborhood V of p_0.*

(ii) *The case where $f(p_0)=+\infty$.* $\lim_{G\ni z\to p_0} H_f^G(z)=f(p_0)$ *holds if* $\iint_{G\cap V} H_{f^-}^G(z)\,dxdy<\infty$ *for a neighborhood V of p_0.*

(iii) *The case where $f(p_0)=-\infty$.* $\lim_{G\ni z\to p_0} H_f^G(z)=f(p_0)$ *holds if* $\iint_{G\cap V} H_{f^+}^G(z)\,dxdy<\infty$ *for a neighborhood V of p_0.*

PROOF. We first suppose f is non-negative. Let $f_n=\min\{f, n\}$, $u_n=H_{f_n}^G$ and $u=H_f^G$, then $u_n\uparrow u$ $(n\to\infty)$. If $f(p_0)=+\infty$, $n=\lim_{G\ni z\to p_0} u_n(z)\leqq \varliminf_{G\ni z\to p_0} u(z)$. Letting $n\to\infty$, we obtain $\lim_{G\ni z\to p_0} u(z)=+\infty=f(p_0)$. If $f(p_0)$ is finite, we denote by α the component of ∂G containing p_0.