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Note on automorphisms in separable extension of non commutative ring

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Preliminaries

All definitions and terminologies in this paper are the same as those in the same author's papers [8], [11] and [13]. So Λ shall be a ring with an identity 1, Γ a subring of Λ which contains 1, C the center of Λ, C' the center of Γ and $\Delta=V_{\Lambda}(\Gamma)=\{x\in\Lambda|xr=rx$ for all $r\in\Gamma\}$. Λ is an H-separable extension of Γ if $\Lambda\otimes_{\Gamma}\Lambda$ is a $\Lambda-\Lambda$-direct summand of some finite direct sum of copies of Λ. In this case Λ is a separable extension of Γ, *i.e.*, map π of $\Lambda\otimes_{\Gamma}\Lambda$ to Λ such that $\pi(x\otimes y)=xy$, for x, $y\in\Lambda$, splits as $\Lambda-\Lambda$-map. As for the fundamental properties of H-separable extension, see [4], [5] and [12]. In [11] and [13] the author showed that in case Γ is a simple artinean ring, Λ is an H-separable extension of Γ if and only if Λ is an inner Galois extension of Γ. It is well known that in this case every automorphism of Λ which fixes all elements of Γ is an inner automorphism. In this paper we will generalize this theorem to the case of ordinal H-separable extensions (Theorem 2). We will also show that every G-Galois extension such that all elements of G are inner automorphisms is an H-separable extension (Theorem 3). For a two-sided Λ-module M, we denote C-submodule $\{m\in M|xm=mx$ for all $x\in\Lambda\}$ by M^{Λ}. Then, Λ is H-separable over Γ if and only if $\Delta\otimes_{C}M^{\Lambda}\cong M^{\Gamma}$ by $(d\otimes m\rightarrow dm)$ (see Theorem 1.2 [8]) for every two sided Λ-module M. We will use this theorem very often throughout this paper. For a ring Λ we denote the Jacobson radical of Λ by $J(\Lambda)$. We will also study in § 3 in what case $J(\Lambda)=\Lambda J(\Gamma)=J(\Gamma)\Lambda$ and $J(\Gamma)=J(\Lambda)\cap\Gamma$ holds when Λ is H-separable over Γ.

1. Automorphisms in *H*-separable extensions.

The first result is a supplement of Theorem 2 [5].

THEOREM 1. *Let Λ be an H-separable extension of Γ. Then every ring endomorphism of Λ which fixes all elements of Γ is an automorphism and fixes all elements of $V_{\Lambda}(V_{\Lambda}(\Gamma))$.*