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On vanishing contact Bochner curvature tensor

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Recently T. Kashiwada [2][1] has given various necessary and sufficient conditions in order that a Kählerian space has vanishing Bochner curvature tensor. In the present paper, we study some conditions in order that a Sasakian space has vanishing contact Bochner curvature tensor and also give some applications of our results.

In § 1 state fundamental identities for the contact Bochner curvature tensor in a Sasakian space.

§ 2 is devoted to the study of conditions in order that a Sasakian space has vanishing contact Bochner curvature tensor and we give two theorems which are analogous to the results due to T. Kashiwada [2].

T. Sakaguchi [4] has introduced the concept of a complex semi-symmetric metric F-connection in a Kählerian space and in terms of certain properties of the connection he has given a sufficient condition in order that a Kählerian space has vanishing Bochner curvature tensor. On the other hand, in a Sasakian space the concept of a contact conformal connection has been introduced by K. Yano [5]. Corresponding to the study of T. Sakaguchi [4], in § 3 we consider a Sasakian space admitting a contact conformal connection. Then, as an application of our first theorem in § 2, we get a sufficient condition in order that a Sasakian space with a contact conformal connection has vanishing contact Bochner curvature tensor.

M. Matsumoto and G. Chuman [3] have studied a compact Sasakian space with vanishing contact Bochner curvature tensor and have given various conditions for the second Betti number to be zero. In § 4, making use of our second theorem in § 2, we show that the theorem of M. Matsumoto and G. Chuman is valid even if one of the conditions in it is replaced by a weaker one.

§ 1. Preliminaries.

Let M be a m-dimensional Riemannian space covered by a system of coordinate neighborhoods $\{(U; y^h)\}$, where here and in the sequel, the indices

1) Numbers in brackets refer to references at the end of the paper.