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On G-functors (I): transfer theorems for cohomological G-functors

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1. Introduction

The purpose of this seuqence of papers is to study and to apply G-functors. The concept of G-functors was introduced by Green [10] during the study of modular representation, particularly a relation between Brauer's theory of blocks and Green's theory of indecomposable modules [9], and it was quite useful for the arrangement of many concepts about representation theory, cohomology theory, etc. Some interesting examples are found in [10], § 5. Afterward, Dress defined the concept Mackey functors which are generalizations of G-functors and applied them to some fields (Dress [3], [4], [5]). Lam's theory also seems to contribute to their theories. These concepts are frequently used rather in equivariant topology, theory of bilinear forms, etc. than in finite group theory itself.

Now, let's observe first the character ring of a finite group G. It is well known that the following theorems about induced characters play important parts in representation theory.

(M) If $H, K\leq G$ and $\alpha\in ch(H)$, then

$$\alpha^{G}{}_{K}=\sum\alpha^{g}{}_{H^{g}\cap K}{}^{K},$$

where g runs over a complete set of representatives of $H\backslash G/K$.

(F) If $H\leq G$, $\alpha\in ch(H)$, $\beta\in ch(G)$, then

$$\alpha^{G}\cdot\beta=(\alpha\cdot\beta_{H})^{G}.$$

The first formula follows from the *Mackey subgroup theorem.* The second means essentially the same fact as the usual *Frobeniud reciprocity.* See, for example, [16], Th. 2. 1, A, B. It is surprising that the formulas (M) and (F) appear also in cohomology theory of finite groups ([2], Prop, 12. 9. 1 ; [19], Prop. 4. 3. 7). The formula (M) is usually called the *double coset for-- mula.* Furthermore the following holds in this case.

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