*Hokkaido Mathematical Journal Vol. 9 (1980) p. 190-216*

# On a parametrix for a weakly hyperbolic operator

By Masato IMAI

(Received November 13, 1979; Revised January 29, 1980)

## § 1. Introduction.

In this paper we consider the Cauchy problem in the domain $[0, T]\times R^{n}$ for the weakly hyperbolic partial differential operator

$$P(t, x, D_t, D_x)=D_t^2+2a(t, D_x)D_t+b(t, D_x)+P_1(t, x, D_t, D_x)\,, \tag{1.1}$$

where $D_t=\dfrac{1}{i}\dfrac{\partial}{\partial t}$ and $D_t=\left(\dfrac{1}{i}\dfrac{\partial}{\partial x_1}, \cdots, \dfrac{1}{i}\dfrac{\partial}{\partial x_n}\right)$.
Here $a(t, D_t)$ and $b(t, D_x)$ are respectively the first and the second order homogeneous partial differential operators depending smoothly on $t$ such that, for any $\xi\in R^{n}$, $a^2(t, \xi)-b(t, \xi)\geqslant 0$ if $t\geqslant 0$. $P_1(t, x, D_t, D_x)$ is an arbitrary first order term with smooth coefficients which are constant for large $|x|$.

Now we impose the following condition for the principal symbol $P_2(t, \tau, \xi)=\tau^2+2a(t, \xi)\tau+b(t, \xi)$ of $P$. For any $(t, \tau, \xi)$, $(\tau, \xi)\neq 0$,

$$grad_{(t,\tau,\xi)}P_2\neq 0\,. \tag{1.2}$$

Note that if $P_2\neq 0$, $grad_{(t,\tau,\xi)}P_2\neq 0$ from the homogeneity of $P_2$. Examples of such $P_2$ are $D_t^2-t\Delta$, $D_t^2-t\Delta_{x'}-\Delta_{x''}$ etc. Here $\Delta$ is the Laplacian. Furthermore $x=(x', x'')$ and $\Delta_{x'}$, $\Delta_{x''}$ are the corresponding Laplacians.

In the following we shall discuss the Cauchy problem

$$\begin{cases} P(t, x, D_t, D_x)\,u=f & \text{in } [0, T]\times R^{n}\,, \\ D_t^j u(0, x)=v_j(x),\ j=0, 1, & \text{in } R^{n}\,, \end{cases} \tag{1.3}$$

for given $f$ and $v_j$. The correctness of (1.3) can be shown by the energy estimate if we reduce $P$ to a simple form by the change of variable of § 2 (see [11]). On the other hand Ivrii discussed the correctness of the Cauchy problem of a weakly hyperbolic operator whose principal symbol has smooth coefficients depending on $(t, x)$ and does not have critical points with respect to $(t, x, \tau, \xi)$ ([7]). He called these operators completely regularly hyperbolic. By the energy estimate, he proved that the Cauchy problem for such an operator is correct, regardless of its lower order terms. The regularity property of its solution does not depend on the lower order terms.

In this paper we shall construct a parametrix of the Cauchy problem