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On Sasakian manifolds with vanishing contact Bochner curvature tensor

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§ 1. Introduction.

Recently, S. I. Goldberg and M. Okumura [3] proved

THEOREM A. *Let M be an n-dimensional compact conformally flat Riemannian manifold with constant scalar curvature R. If the length of the Ricci tensor is less than $R/\sqrt{n-1}$, $n\geqq 3$, then M is a space of constant curvature.*

For a Kaehlerian manifold, Y. Kubo [7] proved

THEOREM B. *Let M be a real n-dimensional Kaehlerian manifold with constant scalar curvature R whose Bochner curvature tensor vanishes. If the length of the Ricci tensor is not greater than $R/\sqrt{n-2}$, $n\geqq 4$, then M is a space of constant holomorphic sectional curvature.*

Note that the square of the length of the Ricci tensor is greater than or equal to R^2/n, so the Ricci tensor has been "pinched".

We have the following remarks [5] on Theorem B.

REMARK 1. The condition with respect to the length of the Ricci tensor can be replaced by

$$(*)\qquad R_{ab}R^{ab}\leqq\frac{R^2}{n-2}\,.$$

REMARK 2. Moreover the condition (*) can be replaced by the best condition

$$R_{ab}R^{ab}<\frac{n^3-2n^2+32}{(n+2)^2(n-4)^2}R^2\qquad\text{for}\quad n>4\,.$$

REMARK 3. In paticular, when M is of dimension 4, if the scalar curvature does not vanish, then M is of constant holomorphic sectional curvature.

The purpose of this paper is to obtain the theorems, analogous to the above theorems, for a Sasakian manifold with vanishing contact Bochner curvature tensor.

THEOREM 1. *Let M be a $(2n+1)$-dimensional Sasakian manifold with*