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On parabolic equations in n space variables and their solutions in regions with edges

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1. Introduction

In this paper we study the initial-Dirichlet problem for parabolic equations of the form

$$Lu=f,$$
$$L=a_{ik}(x)\frac{\partial^{2}}{\partial x_{i}\partial x_{k}}+a_{i}(x,t)\frac{\partial}{\partial x_{i}}+a(x,t)-\frac{\partial}{\partial t}. \qquad (1.1)$$

Here f depends on $x=(x_1, \cdots, x_n)$ and t, and we use the *summation convention* (summations from 1 to n).

Equation (1.1) will be considered in a region $\Omega=G\times J\subset \boldsymbol{R}^{n+1}$, where $J=\{t|0<t\leqq T\}$ and $G\subset \boldsymbol{R}^n$ has edges satisfying conditions to be specified below. L is assumed to have $C^{\alpha}(\bar{\Omega})$-coefficients, where $0<\alpha<1$, and $f\in C^{\alpha}(\bar{\Omega})$, too.

We shall prove that, under these assumptions and suitable conditions concerning the initial and boundary data, for bounded solutions u of that problem we have $D_x u\in C^{\nu}(\bar{\Omega})$, where $0<\nu<1$ and D_x denotes partial differentiation with respect to any x_i, $i=1, \cdots, n$. Also $D_x^2 u\in C^0(\bar{\Omega})$ under an additional assumption.

Our method is based on Schauder type estimates and barrier functions, and the results will extend those in [3] for $n=2$.

Furthermore, it is interesting to note that the method can be modified so that it yields similar results for bounded solutions of the Dirichlet problem for *elliptic* equations in n-dimensional regions with edges. This will be explained at the end of this paper.

We mention that an early paper on regions with edges was T. Carleman's thesis [4] for the n-dimensional Laplace equation. Mixed boundary value problems in two-dimensional regions with corners were also considered by N. M. Wigley [10]. Systems of the form $\Delta u=F(x, u, \text{grad } u)$ in such regions were recently studied by G. Dziuk [5], who obtained results on the smoothness of solutions. Publications on elliptic equations with $n=2$ in regions