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A note on symmetric codes over $GF(3)$

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Let q be a prime power such that $q\equiv 2 \pmod 3$ and $q\equiv 1 \pmod 4$, $GF(q)$ a field of q elements and μ the quadratic character of $GF(q)^{X}$ with $\mu(0)=0$.

Let T be a matrix of degree q defined by $T(a, b)=\mu(b-a)$, where $a, b\in GF(q)$, and

$$S=\begin{pmatrix} 0 & 1\cdots 1 & 1 \\ 1 & & \\ \vdots & T & \\ 1 & & \\ 1 & & \end{pmatrix}.$$

Let $C(q)$ be the code generated by (I, S) over $GF(3)$, which is introduced by V. Pless in [3] and I denotes the identity matrix of degree $q+1$.

The purpose of this note is to show that the minimum weight of $C(q)$ is not smallet than $\sqrt{q}$.

§ 1. Let $C^{*}(q)$ be the code generated by (I, S) over $GF(3^{2})$. Let i be a primitive fourth root of unity in $GF(3^{2})$. Then we may choose

$$\begin{pmatrix} -I-iS, & iI-S \\ -I+iS, & -iI-S \end{pmatrix}$$

as generators of $C^{*}(q)$, since $(-S, I)$ is contained in $C(q)$ (See [3]). We notice that $-i(-I-iS)=iI-S$ and $i(-I+iS)=-iI-S$. Let U and L be the subcodes of $C^{*}(q)$ generated by $(-I-iS, iI-S)$ and $(-I+iS, -iI-S)$ respectively. Then any codevector of $C^{*}(q)$ has a form $(x+y, -i(x-y))$, where $(x, -ix)\in U$ and $(y, iy)\in L$.

LEMMA 1. *Let w denote the weight function. Then we have that*

$$w\Big(x+y,\ -i(x-y)\Big)\geqq w(x) \text{ and } w(y).$$

PROOF. We may label elements of $GF(3^{2})$ as follows: $a_{1}=0$, $a_{2}=1$, $a_{3}=-1$, $a_{4}=i$, $a_{5}=i+1$, $a_{6}=i-1$, $a_{7}=-i$, $a_{8}=-i+1$ and $a_{9}=-i-1$. Now