Hokkaido Mathematical Journal Vol. 10 (1981) p. 83-92

Isometric imbedding of a compact orientable flat Riemannian manifold into Euclidean space

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(Received March 24, 1980)

1. Introduction. The well-known integral formula of Minkowski was generalized by C. C. Hsiung to integral formulas of a closed hypersurface in an $(n+1)$-dimensional Euclidean space [2]. Y. Katsurada and H. Kojyô obtained integral formulas of a more general case, namely of a closed submanifold in a Riemannian manifold [4]. These integral formulas are used to characterize some closed hypersurfaces or submanifolds. For example we have the following theorem [8]

THEOREM A. *Let M be a closed convex hypersurface of a Euclidean space E^{n+1} and M_l the l-th mean curvature. If M_l is constant for an integer l, $1\leqq l\leqq n-1$, then M is totally umbilical, hence a sphere.*

Theorems which take the place of the above theorem in the case of a closed hypersurface and a closed submanifold in a Riemannian manifold were obtained by M. Tani [7] and Y. Katsurada [3] respectively.

In [8] Newton's formulas and some related formulas are used. These formulas were first used by M. Konishi (her name was M. Tani at that time) in [7]. In the present paper we derive them by a simpler method and then use Yano's method in [8] after a slight modification to get some integral formulas for a closed submanifold and finally to get a theorem similar in some respect to Theorem A. In our theorem, however, the Riemannian connection induced on the submanifold is assumed to be flat and our aim is to get a necessary and sufficient condition of the submanifold to lie on some hypersphere of the ambient Euclidean space.

In § 2 we prove Newton's formulas and their derived ones by using a generating function and get some formulas which may be useful in various cases. In § 3 we prove the main theorem. In the present paper we always use the following technique. When H is an n-matrix valued function on M with only real eigenvalues, we define $H(c)=H+cE$ where E is the unit n-matrix and c is a constant such that all eigenvalues of $H(c)$ are positive on M. In § 4 this technique is applied to a closed hypersurface so that we can take off the convexity condition in Theorem A. This section does