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On Amitsur cohomology of rings of algebraic integers

Dedicated to Professor G. Azumaya on his 60th birthday

By Akira HATTORI

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In connection with the study of Azumaya algebras over rings, we introduced in [4] certain Amitsur-type cohomology groups $\boldsymbol{H}^q(S/R)$ for an extension S/R of commutative rings. The present article is a supplement to that paper, and deals with special features of groups $\boldsymbol{H}^q(S/R)$ in arithmetical context.

As is the case for the groups $\boldsymbol{H}^q(S, G)$ of group cohomology-type [3], [6], we can apply the device of mapping cones to the construction of groups $\boldsymbol{H}^q(S/R)$, thus dispensing with the intermediary of the whole category of invertible modules. This is done in § 1, based upon the general foundations in [6] § 1. In § 2 we deal with local fields, and in § 3 global fields, where we proceed almost parallel to [3] § 6. Parallel though they are, the results are not the same since, roughly speaking, $\boldsymbol{H}^q(S/R)$ almost ignores the ramification, while $\boldsymbol{H}^q(S, G)$ is essentially involved with it. The relationship between these two series of cohomology groups is studied to some extent in [5], but remains to be further clarified. As an example, we show in the final § 4 that for the integer rings of imaginary quadratic fields the unit-valued Amitsur cohomology vanishes in every dimension.

§ 1. Groups $\boldsymbol{H}^q(S/R)$ via mapping cone

1.1. Let R be a commutative ring (with unity). Let F be a covariant functor from the category of commutative R-algebras to the category of abelian groups. Denote the Amitsur's complex of F concerning an R-algebra S by $\mathrm{Am}(S/R, F)$, and its cohomology groups by $H^q(S/R, F)$. (F need not be meaningful on the whole category of R-algebras. It is only required that R is defined in a subcategory sufficient to work with Amitsur cohomology.) A morphism of functors $f: F \rightarrow F'$ yields a complex morphism $\mathrm{Am}(S/R, F) \rightarrow \mathrm{Am}(S/R, F')$. We denote the *mapping cone* of this morphism by $\mathrm{Am}(S/R, f)$. Hence we have an exact sequence

$$0 \longrightarrow \mathrm{Am}(S/R, F') \longrightarrow \mathrm{Am}(S/R, f) \longrightarrow \mathrm{Am}(S/R, F)_{\#} \longrightarrow 0$$

where $C_{\#}$ for a cochain complex C is defined by $C_{\#}^q = C^{q+1}$, $d_{\#}^q = -d^{q+1}$.