Hokkaido Mathematical Journal Vol. 10 (1981) p. 1–12

Finitely generated projective modules over hereditary noetherian prime rings

Dedicated to Professor Goro AZUMAYA on his 60th birthday

By Kenji NISHIDA

(Received February 20, 1980; Revised March 22, 1980)

Introduction. In this paper we study finitely generated projective modules over a hereditary noetherian prime ring (henceforth, we denote an HNP ring, for abbreviation). We mainly concern with the genus of finitely generated projective modules. When one deals with an order over a Dedekind domain, the genus can be investigated by localization (cf. [7, § 27, 35]). In our (noncommutative) case, the localization at a maximal invertible ideal studied in [5, § 3] is very useful. As is stated in [5, § 3], the localization of an HNP ring at a maximal invertible ideal is either a Dedekind prime ring or a semilocal HNP ring defined in § 1. Although finitely generated projective modules over a Dedekind prime ring were perfectly studied in [1], the another case is not treated anywhere. Therefore, we investigate those over a semilocal HNP ring in § 1 and give a necessary and sufficient condition when two finitely generated projective modules are in the same genus and also prove the following.

(1. 15) THEOREM. *Let R be a semilocal HNP ring with its radical I and M, N, K finitely generated projective modules in the same genus such that K/KI contains all $S_\lambda \in \mathfrak{S}_I$. Then there exists a finitely generated projective module L in the genus such that $N \oplus K \cong M \oplus L$.*

In § 2, we treat of an HNP ring R with enough invertible ideals and show that two finitely generated projective modules are in the same genus iff their localization M_I and N_I are in the same genus as R_I-modules for all maximal invertible ideals I, where R_I is the localization of R at I. The generalization of (1. 15) is obtained in (2. 7).

Finally, in § 3, applying the above results we try to define the ideal class group for some HNP ring.

Throughout this paper, R is an HNP ring which is not artinian and Q is the maximal quotient ring of R. We shall shortly mention definitions and notation which will be frequently used in this paper. For more detailed