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Some considerations on various curvature tensors

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K. Yano has introduced the notion of complex conformal connections in Kählerian spaces and showed

THEOREM A ([12]). *In a Kählerian space of real dimension* $\geqq 4$, *if there exists a complex conformal connection with zero curvature, then the Bochner curvature tensor of the space vanishes.*

K. Yano has also introduced the notion of contact conformal connections in Sasakian spaces corresponding to complex conformal connections in Kählerian spaces and had

THEOREM B ([13]). *In a Sasakian space of dimension* $\geqq 5$, *if there exists a contact conformal connection with zero curvature, then the contact Bochner curvature tensor of the space vanishes.*

In the present paper, we consider the converses of Theorem A and Theorem B.

We give algebraic preliminaries and notations in §§1 and 2. §3 is devoted to the proof of Theorem 1, which asserts that if there exists a non-constant solution of a certain partial differential equation, the converse of Theorem A is true. In §4, from a viewpoint of the notions of K-curvature and F-invariant curvature tensors, we define the Bochner curvature tensor of a K-space. Theorem 2 gives a characterization of the vanishing of the Bochner curvature tensor of a K-space. Lemma 10 shows that the converse of Theorem B is true, if there exists a non-constant function satisfying a certain system of partial differential equations.

We remark that a Sasakian space satisfying the assumptions in Lemma 10 admits another Sasakian structure of constant ϕ-holomorphic sectional -3. The latter part of §5 is devoted to the study of a system of partial differential equations in Lemma 10. Theorem 3 and Theorem 4 give a characterization of the Sasakian structure of constant ϕ-holomorphic sectional curvature -3.

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Throughout this paper, our arguments are local and sometimes pointwise.