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Classification of cubic forms with three variables

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Introduction

A degree 3 homogeneous polynomial, $\gamma=\sum_{1\leqq i,j,k\leqq n}a_{ijk}x_{i}x_{j}x_{k}$ is called a cubic form. Our objective is to classify the set of cubic forms by linear translations. Generally, let f be a singular germ with an isolated critical point at origin and corank n. Form the Thom's splitting lemma (D. Gromoll and W. Meyer [3]), f is right equivalent to $g+Q$ where $g(x_{1}, x_{2}, \cdots, x_{n})\in m^{3}$ and $Q(x_{n+1}, x_{n+2}, \cdots, x_{n+k})$ is a nondegenerate quadratic form. Therefore it is fundamental to give the information of canonical form of 3-jet of g, when we classify the finitely determined singular germ. Indeed, D. Siersma [6] classifies the singularities with the right codimension $\leqq 8$. In his paper, one of the difficulties of the classification is the canonical form of 3-jet g, though results of algebraic geometry and the work of Mather [4] (G. Wassermann [7]) are widely used.

In this paper, we will try to classify cubic forms with 3-variables. Our conclusion coincides with the work of van der Waerden [7] concerning with the surfaces represented by cubic forms that is the curves represented by cubic forms in the projective plane. The main result is theorem 4.1. We shall prove the theorem 4.1 in terms of the concepts of homology and intersection theory in manifolds. We give a proof in § 4, in which theorem 3.2 is crucial. It is very likely that the theorem also holds for $n>3$. At the end, I would like to thank Professor H. Suzuki and Professor Fukuda for their helpful advices.

§ 1. Preparation

Let $S(n)$ be the set of all $(n\times n)$-symmetric matrices and $SL(n)$ the special linear group. We define an $SL(n)$-action on $S(n)$ by setting $F_{P}A=PAP'$ for $A\in S(n)$, $P\in SL(n)$, where P' is the transposed matrix of P. Denote by $G_{k}(S(n))$ the set of k-dimensional linear subspaces of $S(n)$ when we view $S(n)$ as a vector space. We define the $SL(n)$-action on $G_{k}(S(n))$ by setting $F_{P}\gamma=\{F_{P}A|A\in\gamma\}$ for $P\in SL(n)$, $\gamma\in G_{k}(S(n))$. This is well defined, for F_{P} is a linear automorphism of $S(n)$ for each $P\in SL(n)$. Let γ