Hokkaido Mathematical Journal Vol. 10 (1981) p. 209-238

Reduction modulo $\mathfrak{P}$ of Shimura curves

By Yasuo MORITA

(Received May 21, 1980)

0-1. Let F be a totally real algebraic number field of finite degree g, and let B be a division quaternion algebra over F such that $B\otimes_{\boldsymbol{Q}}\boldsymbol{R}$ is isomorphic to the product of $M_2(\boldsymbol{R})$ and $g-1$ copies of the division quaternion algebra $\boldsymbol{H}$ over $\boldsymbol{R}$. Let G be the algebraic F-group satisfying $G_F=B^{\times}$, let G_A be the adelization of G, and let G_{A+} be the subgroup of G_A consisting of all elements whose projections to $M_2(\boldsymbol{R})$ have positive determinants. Let $G_{\infty+}$ and G_0 be the archimedean part and the finite part of G_{A+}, let $G_{\boldsymbol{Q}+}=G_{A+}\cap G_F$, and let $\mathcal{Z}$ be the family consisting of all subgroups S of G_{A+} such that S has the form $S=G_{\infty+}\cdot S_0$ with an open compact subgroup S_0 of G_0.

For each $S\in\mathcal{Z}$, let $\Gamma_S=S\cap G_{\boldsymbol{Q}+}$, and we regard Γ_S as a subgroup of $GL(2, \boldsymbol{R})$. Then Γ_S acts on the complex upper half plane $\mathfrak{H}$ in the usual way, and $\Gamma_S\backslash\mathfrak{H}$ is a complete non-singular curve. Let ν be the reduced norm of B, and let k_S be the abelian extension of F corresponding to the subgroup $\nu(S)\cdot F^{\times}$ of $F_A^{\times}$ by class field theory. Then Shimura constructed an algebraic curve V_S defined over k_S and a holomorphic map φ_S of $\mathfrak{H}$ onto V_S inducing $\Gamma_S\backslash\mathfrak{H}\cong V_S$, satisfying certain algebraic and arithmetic conditions (cf. 1-1).

Let p be a prime number, and let $\mathfrak{P}$ be an extension of p to a place of $\overline{\boldsymbol{Q}}$. Then we shall show that *V_S has good reduction at $\mathfrak{P}$ if* (i) *$\mathfrak{P}$ does not divide the discriminant $D(B/F)$ of B and* (ii) *the "level" of S is prime to p.* (For the exact statement, see Main Theorem 1 in 1-2.) Furthermore, as was conjectured in Shimura [24], 2. 9, we shall *construct a system of curves over finite fields* satisfying several conditions (see Main Theorem 3).

0-2. The exact statements of our main results are in § 1. The proof starts in § 2 and ends in § 3.

In 1-1, we quote the result of Shimura [24] in our case. In 1-2, the main results are stated. In 1-3, a summary of the proof of Shimura's result is given. In 2-1, we quote from Mumford [14] the existence of the fine moduli scheme for polarized abelian schemes with level structures. In 2-2 and 2-3, we construct moduli spaces for families of PEL-sturctures by