Hokkaido Mathematical Journal Vol. 10 (1981) p. 333-342

Bimodule structure of certain Jordan algebras relative to subalgebras with one generator

By N. JACOBSON*

Dedicated to Goro Azumaya on his sixtieth birthday

(Received October 17, 1980)

Throughout this paper "algebra" will mean finite dimensional algebra with unit over a field F and, unless otherwise indicated, "algebra" without modifier will mean associative algebra. An algebra is called a *Frobenius algebra* if there exists a non-degenerate associative bilinear form $f(x, y)$ on A, where associativity means that

$$f(ab, c)=f(a, bc) \tag{0.1}$$

for a, b, $c\in A$. This condition is readily seen to be equivalent to: A contains a hyperplane that contains no non-zero one sided ideal.

A number of years ago we proved the following result on generation of central simple algebras.

0.1. THEOREM. *Let A be a central simple algebra of degree n, C a commutative Frobenius subalgebra of n dimensions. Then A contains an element b such that $A=CbC$ (Jacobson* [1]).

It is well known that an algebra with a single generator is Frobenius (see e.g. Jacobson [1], p. 219). Hence we have the following consequence of this theorem.

0.2. COROLLARY. *Let A be a central simple algebra of degree n, a an element of A such that $[F[a]:F]=n$. Then A contains an element b such that $A=F[a]\,bF[a]$.*

The proof of Theorem 0.1 is based on the following facts:

1. The tensor product of Frobenius algebras is Frobenius. 2. If C is a commutative Frobenius algebra then any faithful representation of C contains the regular representation as a direct component. 3. If B is a subalgebra of a central simple algebra then A regarded as a bimodule for B in the natural way can be regarded as a faithful module for $B\otimes B^{op}$. This follows from the fact that A is faithful as $A\otimes A^{op}$ module which in turn follows from the simplicity of $A\otimes A^{op}$.

* This research was partially supported by the National Science Foundation grant MCS 79-04473.