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Blocks with a normal defect group*

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To Professor Goro Azumaya to commemorate his sixtieth birthday

1. Introduction.

Let G be a finite group, p a fixed rational prime and P be a Sylow p-subgroup of G. In the following, we will consider the group algrbeas over a complete discrete valuation ring R with the unique maximal ideal $(\pi)\ni p$, where its residue class field $F=R/(\pi)$ of characterist c p is a splitting field for G.

In this paper we shall introduce two invariants $n(B)$, $m(B)$ (both positive integers) which can be associated with a given p-block B of G. Namely, $n(B)$ is the number of indecomposable direct summands of $B_{P\times P}$ (the restriction of a $G\times G$-modules B to $P\times P$), and $m(B)$ is the number of indecomposable direct summands of $B_{\Delta(P)}$, where Δ is the diagonal map from G to $G\times G$. These ideas are derived from module-theoretic concepts of a block ideal B which is due to works of J. A. Green ([10], [11], [13]).

On the other hand, Brauer investigated the relation between the invariants $k(B)$, $l(B)$ (the number of ordinary and modular irreducible characters in B, respectively) and the integer $v(B)$ defined by

$$\dim B=p^{2a-d}v(B),$$

where $p^{a}=|P|$ and d is the defect of B (see section 2 in this paper and Brauer [5]). Following R. Brauer, we shall obtain an elementary inequality

$$\text{(2E, 1)} \qquad p^{a-d}v(B)\leqq m(B)\leqq p^{a}n(B)\leqq p^{a}v(B).$$

Our main interest is in the "extreme" cases of (2E, 1), namely,

$$n(B)=v(B) \quad \text{and} \quad m(B)=p^{a}v(B).$$

Then, in section 3, we will give the structure of G in the above cases. Consequently, for example, it is proved that if $B=B_{0}$, the principal block, then $n(B_{0})=v(B_{0})$ if and only if $G=O_{p'pp'}(G)$, and $m(B_{0})=p^{a}v(B_{0})$ if and only if $G=O_{p'p}(G)$ and P is abelian. In section 4, we will consider another

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