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Finitely generated projective modules over hereditary noetherian prime rings II

Dedicated to Professor Kentaro MURATA
on his 60th birthday

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The purpose of this paper is to generalize [5, Theorem (2.6)] as the following.

THEOREM. *Let R be a hereditary noetherian prime ring and M, N finitely generated projective modules such that $N\subset M$ and* rank $M=$ rank N. *Let $N=N_0\subset N_1\subset\cdots\subset N_n=M$ be a composition series of M/N, $S_i=N_i/N_{i-1}$ $(i=1, \cdots, n)$, $\mathscr{S}=\{S_i; i=1, \cdots, n\}$, and $\mathscr{P}=\{P; P$ is an idempotent maximal ideal such that $S_iP=0$ for some $S_i\in\mathscr{S}\}$. Then $M\sim N$ iff the following hold;*

1) *for an idempotent maximal ideal $P\notin\mathscr{S}$ and a simple right R-module S with $SP=0$, $\mathrm{Ext}_R^1(S_i, S)=0$ for every faithful simple module $S_i\in\mathscr{S}$,*

2) *for an idempotent maximal ideal $P\in\mathscr{P}$ which belongs to a cycle $\{P_1, \cdots, P_k\}$, $\mathscr{S}$ includes each simple right R-module T_j with $T_jP_j=0$ $(j=1, \cdots, k)$ by the same number,*

3) *for an idempotent maximal ideal $P\in\mathscr{P}$ which belongs to a strictly open cycle $\{P_1, \cdots, P_k\}$, $\mathscr{S}$ includes each simple right R-module T_j with $T_jP_j=0$ $(j=1, \cdots, k)$ and a faithful simple right R-module T with $\mathrm{Ext}_R^1(T, T_k)\neq 0$ by the same number.*

Throughout the paper, let R be a hereditary noetherian prime ring and M, N finitely generated projective right R-modules. M and N are said to be of the same *genus* [3], denoted by $M\sim N$, if rank $M=$ rank N and $M/MP\cong N/NP$ for all maximal ideals P of R. In the previous paper [5], we studied the condition for $M\sim N$ when R has enough invertible ideals. We shall extend a portion of [5] to the general case.

Let Q be the maximal quotient ring of R. For a fractional R-ideal I, we put $O_r(I)=\{x\in Q; Ix\subset I\}$ and $O_l(I)=\{x\in Q; xI\subset I\}$. A finite set of distinct idempotent maximal ideals $\{P_1, \cdots, P_k\}$ of R is called a *cycle* if $O_r(P_i)$