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Linear parabolic equations in regions with re-entrant edges

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In a recent paper [1] we studied solutions of the parabolic equation

$$(1)\qquad Lu(x, t)=a_{ik}(x)u_{x_{i}x_{k}}+a_{i}(x, t)\,u_{x_{i}}+a(x, t)\,u-u_{t}=f(x, t)\,,$$

$x=(x_{1}, \cdots, x_{n})$, in a simply connected, bounded region $\Omega=G\times J\subset \boldsymbol{R}^{n+1}$, $n\geqq 2$, $J=\{t|0<t\leqq T\}$, satisfying the conditions

$$(2\text{ a})\qquad u(x, 0)=0,\ x\in\bar{G}\,,$$

$$(2\text{ b})\qquad u\Big|_{\partial G\times\bar{J}}=\phi(x, t)\,,$$

under the following assumptions.

(A) $a_{ik}\in C^{\alpha}(\bar{G})$, a_{i}, a, $f\in C^{\alpha}(\bar{\Omega})$, $0<\alpha<1$,

(B) $\phi(x, 0)=0$, $\phi\in C^{2+\alpha}[\partial G\backslash E)\times\bar{J}]\cap C^{0}(\partial G\times\bar{J})$,

(C) $\omega(P)<\pi$ for all $P\in E$.

Here $E=\bigcup_{i} E_{i}$, where $E_{1}, \cdots, E_{m}$ are $(n-2)$-dimensional edges (the intersections of portions of hypersurfaces $\Gamma_{1}, \cdots, \Gamma_{m}$ constituting the boundary ∂G of G), and $\omega(P)$ is the angle between the images of the two Γ_{j}'s corresponding to a point P: x^{0} of E under the transformation of

$$a_{ik}(x^{0})\,u_{x_{i}x_{k}}^{*}=0$$

to canonical form.

Condition (C) means that the edges of the image of G are non-reentrant. The question arises whether this restriction can be removed. This would be of practical importance, for the following reason.

It is well known that in physical and other applications, a great majority of boundary value or initial value problems are such that the given data or the boundaries of the domains have singularities (corners or edges); cf. [3], Chaps. V, VI, [4], [10], [13]. Not infrequently, some of those edges are re-entrant; for typical examples, see [5], Chap. 3, [7], Secs. 24.3-24.7, [8], Chap. 24, and [14], Chap. 8. In each such case, it is desirable to have knowledge about the kind of singularities of solutions and derivatives one has to expect, as a consequence of the singularities of the boundary.

The knowledge just mentioned is even more mandatory in finite differ-