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Solvability of some groups

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Introduction. K. Nomura posed the following problem: Let G be a finite group that has a large inner automorphism consition. Then, is G a solvable group? Especially, we call a finite group G to be an AI-group if G satisfies the following: $N_G(A)/C_G(A)$ is isomorphic to the full automorphism of A for every Abelian subgroup A of G.

The purpose in the paper is to show the following theorem:

THEOREM A. *Let G be a finite AI-group, then G is solvable.*

To prove the theorem, we introduce the weaken condition. A finite group G is called an A_3I-group if G satisfies the following condition: For every Abelian $3'$-subgroup A and a 3-subgroup B of $C_G(A)$, $C_G(B)\cap N_G(A)/C_G(B)\cap C_G(A)\geqq O^3(\mathrm{Aut}(A))$. We will say that $C_G(B)$ covers $O^3(\mathrm{Aut}(A))$ if the condition holds.

Using the above notion, we will change the form of the theorem.

THEOREM B. *The following hold*:

a) *All AI-groups are A_3I-groups.*

b) *Every A_3I-group is solvable.*

Clearly, Theorem A is an immediate consequence of Theorem B. Most of our notation is standard and taken from [1]. All groups considered in this paper will be finite. Let G be a group. Then $F(G)$ denotes the Fitting subgroup of G.

2. Preliminary lemmas.

In this section, we will search the properties of A_3I-groups.

LEMMA 2.1. *Let G be a finite A_3I-group and B be a 3-subgroup of G. Then the following hold*;

a) *$i(G)=1$, the conjugate class of involutions of G is one.*

b) *$Z(P)$ is of order p for a Sylow p-subgroup P of G and every prime divisor p of the order of G, $(p\neq 3)$.*

c) *$C_G(B)$ is an A_3I-group.*

d) *$G/O_3(G)$ is an A_3I-group.*

e) *$O^3(G)$ is an A_3I-group.*

PROOF. By the definition of A_3I-groups, we easily get the above state-