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On irreducible conformally recurrent Riemannian manifolds

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Recently, A. Gębarowski [1] has determined decomposable conformally recurrent Riemannian manifolds. Then, it is natural to ask how irreducible conformally recurrent Riemannian manifolds are determined. In fact, taking account of the de Rham decomposion of Riemannian manifolds, a complete simply connected Riemannian manifold is either irreducible or decomposable. It is natural to assume that the Riemannian manifold has a complete metric and is simply connected by considering its universal covering space, if necessary.

The purpose of the present paper is to show

THEOREM. *An analytic irreducible conformally recurrent Riemannian manifold of dimension greater than 4 is conformally flat.*

§ 1. Preliminaries and notation.

Let M be an $n(\geqq 4)$-dimensional Riemannian manifold with a positive definite Riemannian metric g_{ji}. We denote by ∇_{j}, $K_{kji}{}^{h}$, K_{ji} and K the operator of covariant differentiation, the Riemannian curvature tensor, the Ricci tensor and the scalar curvature, respectively. Then the conformal curvature tensor C_{kjih} is given by

$$(1)\qquad C_{kjih}=K_{kjih}-\frac{1}{n-2}(K_{kh}g_{ji}-K_{jh}g_{ki}+g_{kh}K_{ji}-g_{jh}K_{ki}) +\frac{K}{(n-1)(n-2)}(g_{kh}g_{ji}-g_{jh}g_{ki}).$$

For the conformal curvature tensor C_{kjih} we know the following identities:

$$(2)\qquad C_{kjih}=-C_{jkih}=-C_{kjhi}=C_{ihkj}, \quad C_{kjih}+C_{jikh}+C_{ikjh}=0,\ g^{kh}C_{kjih}=0,$$

$$(3)\qquad \nabla_{h}C_{kji}{}^{h}=\frac{n-3}{n-2}C_{kji},\ \text{where } C_{kji}=\nabla_{k}K_{ji}-\nabla_{j}K_{ki} -\frac{1}{2(n-1)}(\nabla_{k}Kg_{ji}-\nabla_{j}Kg_{ki}),$$