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On vanishing or recurrent Bochner curvature tensor

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The Bochner curvature tensor has been introduced by S. Bochner ([2]) in Kaehlerian manifolds with respect to the complex coordinates, as an analogy of the conformal curvature tensor. S. Tachibana gave it a real expression ([12]) and showed that a reducible Kaehlerian manifold M with vanishing Bochner curvature tensor is locally the Riemannian product of two Kaehlerian manifolds with constant holomorphic sectional curvatures $H(\geqq 0)$ and $-H$, respectively, and in this case the scalar curvature of M is constant ([13]). On the other hand, since it is known ([8]) that a Kaehlerian manifold M with vanishing Bochner curvature tensor and constant scalar curvature is locally symmetric, if M is irreducible, then M is Einsteinian and thus of constant holomorphic sectional curvature. Therefore, taking account of de Rham decomposition of Kaehlerian manifolds ([6], II), we can determine complete (simply connected) Kaehlerian manifolds with vanishing Bochner curvature tensor and constant scalar curvature. In the present paper, we shall show, by using the theory of fibred spaces with projectable Kaehlerian structure of M. Ako ([1]), that a real $n(\geqq 4)$-dimensional Kaehlerian manifold with vanishing Bochner curvature tensor has constant scalar curvature.

Recurrent geometry has been introduced by A. G. Walker ([14]) and he has determined recurrent manifolds, that is, Riemannian manifolds with recurrent Riemannian curvature tensor (see also [6], I). Projective recurrent manifolds, that is, Riemannian manifolds with recurrent projective curvature tensor, were determined by M. Matsumoto ([7]). Conformally recurrent manifolds, that is, Riemannian manifolds with recurrent conformal curvature tensor, were determined by A. Gebarowski ([4]) in the case that the mainfolds are reducible, and by [9] in the case that the manifolds are irreducible and of dimension greater than four. In the present paper, we shall study Kaehlerian manifolds with recurrent Bochner curvature tensor and give some results in reducible and irreducible cases (Theorems 3 and 4).

§ 1. Preliminaries.

(I) Fibred spaces. Let M and N be two manifolds of dimensions n