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Ideals of hereditary noetherian prime rings

Dedicated to Professor Kentaro Murata
on his 60th birthday

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Introduction. Let R be a hereditary noetherian prime ring (abbr. HNP ring), and let I be an ideal of the form $M_1\cap\cdots\cap M_k$, where $M_1, \cdots, M_k$ are distinct idempotent maximal ideals of R with $O_r(M_1)=O_l(M_2), \cdots, O_r(M_{k-1})=O_l(M_k)$. In the study of ideals of HNP rings, it is important to consider such ideals (cf. [2]). When $M_1, \cdots, M_k$ form a cycle (i. e., moreover $O_r(M_k)=O_l(M_1)$ holds), I is an invertible ideal and the properties of such ideals were broadly studied in [2, 6]. In this paper, we present some properties of the ideal I when $M_1, \cdots, M_k$ form an open cycle (i. e., moreover $O_r(M_k)\neq O_l(M_1)$ holds (Theorem 1. 3). We also give the structure of an eventually idempotent ideal (Theorem 1. 4), and minimal idempotent ideals provided R has finitely many idempotent maximal ideals (Theorem 1. 5). We consider in section 2 an idealizer C of an HNP ring R and completely determine all maximal ideals of C and their relations stated by their maximal right (left) orders (Proposition 2. 2, Theorems 2. 4-6). By this we can give an example of an HNP ring which has finitely arbitrary many strictly open cycles of 'arbitrary size' (Corollary 2.8).

A part of Theorem 1. 3 has been independently obtained by S. Singh [8], however, inasmuch as our proof is not only different from his but also interesting itself, we shall present it in our context.

Throughout this paper, R is an HNP ring which is not artinian and Q is its maximal quotient ring. For submodules A, B of Q, we put $A^{\cdot}.B=\{q\in Q;\ Aq\subset B\}$, $B.^{\cdot}A=\{q\in Q;\ qA\subset B\}$, $O_r(A)=\{q\in Q;\ Aq\subset A\}$, and $O_l(A)=\{q\in Q;\ qA\subset A\}$. An ideal I of R is *invertible* (resp. *idempotent*) if $I(I^{\cdot}.R)=R=(R.^{\cdot}I)I$ (resp. $I=I^2$). As concerns the properties of HNP rings, the reader is referred to [2, 6, 7].

1. Idempotent ideals. A finite set of distinct idempotent maximal ideals $M_1, \cdots, M_k$ of an HNP ring R is called an *open cycle* (resp. *cycle*) if $O_r(M_k)\neq O_l(M_1)$ (resp. $O_r(M_k)=O_l(M_1)$) and $O_r(M_i)=O_l(M_{i+1})$ for $i=1, \cdots, k-1$. An open cycle $\{M_1, \cdots, M_k\}$ is *right* (resp. *left*) *strictly open* if $O_r(M_k)$