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On H-separable extensions of two sided simple rings

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§1. **Introduction.** Throughout this paper A is a ring with the identity 1, and B is a subgring of A such that $1\in B$. Each B-module (or A-module) is unitary, and each $A-A$-module M satisfies that $(am)b=a(mb)$ for a, $b\in A$ and $m\in M$. In addition we will set $C=V_{A}(A)$, the center of A, and $D=V_{A}(B)$ the centralizer of B in A

We say that A is an H-separable extension of B in the case where ${}_{A}A\otimes_{B}A_{A}<\oplus_{A}(A\oplus A\oplus\cdots\oplus A)_{A}$ (direct summand of a finite direct sum of copies of A). As for some characterizations and properties of H-separable extension see for example [3], [6], [9] and [10].

In this paper we will deal with H-separable extensions of two sided simple rings. In particular, in the case where B is a two sided simple ring we will show that A is right B-finitely generated projective and an H-separable extension of B, if and only if A is a two sided simple ring, $V_{A}(V_{A}(B))=B$ and $V_{A}(B)$ is a simple C-algebra (Theorem 1). Furthermore, under the conditions of Theorem 1 we will show that for any simple C-subalgebra T of D, $V_{A}(T)$ is two sided simple, $V_{A}(V_{A}(T))=T$ and A is an H-separable extension of $V_{A}(T)$ and right $V_{A}(T)$-finitely generated projective (Proposition 2). Finally, under the same conditions we will obtain a duality on two sided simple subrings, which is similar to the well known classical inner Galois theory on simple (artinian) rings (Theorem 2).

§2. We say that A is a two sided simple ring in case A has no proper two sided ideal except 0, and a right artinian two sided simple ring with 1 is called a simple ring. Whenever we call A a simple algebra over a field K, A shall be a K-algebra which is two sided simple and $[A:K]<\infty$. Hereafter we will call each two sided ideal simply an ideal.

Given a right A-module M, set $\Omega=\mathrm{Hom}(M_{A}, M_{A})$. Then, as is well known, M is an $\Omega-A$-module, and we have an $A-A$-map

$$\tau:\ \mathrm{Hom}(M_{A}, A_{A})\otimes_{\Omega}M\longrightarrow A$$

such that $\tau(f\otimes m)=f(m)$ for $f\in\mathrm{Hom}(M_{A}, A_{A})$ and $m\in M$. $\mathrm{Im}\,\tau$ is an ideal of A, and $\mathrm{Im}\,\tau=A$ if and only if M is a right A-generator. Therefore if A is two sided simple and $\mathrm{Hom}(M_{A}, A_{A})\neq 0$, we have $0\neq\mathrm{Im}\,\tau=A$. Thus