Hokkaido Mathematical Journal Vol. 13 (1984) p. 92-108

On separation points of solutions to Prandtl boundary layer problem

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(Received October 8, 1983)

1. Introduction

The equations for 2-dimensional stationary boundary layer theory of incompressible fluid past a rigid wall are

$$\text{(1.1)} \qquad \begin{aligned} uu_x + vu_y &= \nu u_{yy} - p_x , \\ u_x + v_y &= 0 \end{aligned}$$

in the domain $D_A = \{(x, y);\ 0 < x < A,\ 0 < y < \infty\}$ (see [1], [8], [9], [10] and [11]). Here the subscripts x and y denote the partial differentiation with respect to the corresponding variable, (x, y) are orthogonal coordinates in the boundary layer with x representing the length along the wall and y the perpendicular distance from the wall, $u = u(x, y)$ and $v = v(x, y)$ are the corresponding unknown velocity components. The constant ν is a viscous coefficient. Finally $p = p(x)$ is a pressure function. Let $U = U(x)$ be an exterior streaming speed; we assume that $p(x)$ and $U(x)$ satisfy the Bernoulli law and the origin $(0, 0)$ is not a stagnation point, i.e.,

$$\text{(1.2)} \qquad \begin{aligned} U(x)\, U_x(x) + p_x(x) &= 0 , \\ U(0) &> 0 . \end{aligned}$$

The appropriate boundary conditions are

$$\text{(1.3)} \qquad \begin{aligned} &u = v = 0 \quad \text{for} \quad y = 0 \quad \text{and} \quad u(x, y) \longrightarrow U(x) \quad \text{as } y \to \infty , \\ &\text{uniformly in } x \text{ on any compact subset of } [0, A) . \end{aligned}$$

In order to obtain a well-set problem, we suppose that at an initial position, say $x = 0$, an initial datum $u_0(y)$ is assigned to the velocity component u, i.e.,

$$\text{(1.4)} \qquad u(0, y) = u_0(y) \qquad (0 \leq y < \infty) .$$

In this paper we study the existence of the separation point of the flow deterministically.

Hereafter, unless otherwise provided, we assume that the datum $u_0(y)$ belongs to $I^{2+\alpha} = I^{2+\alpha}(\nu, U)$ (for notations see Section 2) and that the speed $U(x)$ and the pressure gradient $p_x(x)$ have following properties: