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A short proof to Brauer's third main theorem

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Introduction

In this note we present a short proof which uses Brauer's first main theorem, Nagao's lemma and some basic results from block thoery.

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1. Notation and basic results

In this paper G is a finite group of order $|G|$, K is a field of characteristic $p>0$ which is algebraically closed (however see the remark at the end) and KG is the group-algebra. Let H be a subgroup of G. $C_G(H)$ and $N_G(H)$ stand for the centralizer and the normalizer of H in G, respectively. Following R. Brauer, we shall call a block b of H admissible if b has a defect group D such that $C_G(D)\subseteq H$.

We recall some definitions and results for convenience.

(a) A block b of H is called the principal block if b contains the trivial representation of H.

(b) The defect groups of the principal block of H and the vertices of the trivial representation are Sylow p-subgroups of H.

(c) ([2, sec. 6]) If B is a block of G with a defect group D and $DC_G(D)\subseteq H$, then H has a block b with a defect group D which satisfies $b^G=B$.

(d) ([2, sec. 6]) Let b_1 and b_2 be admissible blocks of H satisfying $b_1^G=b_2^G$. If H is normal in G then b_1 is conjugate to b_2 in G.

(e) ([1, 57.4, 58.3]) Let b be a block of H with a defect group D. If b^G is defined then it has a defect group which contains D.

(f) Brauer's first main theorem ([1, 65.4]). Let D be a p-subgroup of G and let $H=N_G(D)$. There exists a one to one correspondence between the blocks of G with a defect group D and the blocks of H with defect