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Remarks on Xia's inequality and Chevet's inequality concerned with cylindrical measures

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§ 1. Introduction

In [5] D. Xia has established a certain inequality concerned with quasi-invariant measures, and thereafter his result was extended to the cylindrical measure case by W. Linde [2] and the author [4]. On the other hand, in [1] S. Chevet has established a similar inequality concerned with kernels of cylindrical measures.

The main purpose of the present paper is to give the generalizations of their results. Explicitly stating, we shall prove the following theorems.

THEOREM 1. 1. *Let E and F be linear topological spaces, T be a continuous linear mapping of F into E, and suppose that F is barrelled. Let f be a function defined on E^* (but not necessarily everywhere finite) which satisfies the following two conditions;*

(1) $0\leqq f(tx^*)\leqq tf(x^*)\leqq\infty$, *for every $t>0$ and every $x^*\in E^*$,*

(2) *for every $y\in F$, there is a $\delta>0$ such that the inequality $f(x^*)<\delta$ implies $|\langle x^*, T(y)\rangle|<1$, for every $x^*\in E^*$.*

Then there exists a neighborhood V of zero in F such that for every $x^*\in E^*$, the inequality

$$\sup_{y\in V}\left|\left\langle x^*, T(y)\right\rangle\right|\leqq f(x^*)$$

holds.

THEOREM 1. 2. *Let E and F be linear topological spaces, T be a continuous linear mapping of F into E, and suppose that F is of the second category. Let $\{f_n\}$ be a sequence consisting of functions defined on E^* (but not necessarily everywhere finite) which satisfies the following two conditions;*

(1) $0\leqq f_n(tx^*)\leqq tf_n(x^*)\leqq\infty$, *for every $t>0$, every natural number n and every $x^*\in E^*$,*

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