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On the group of isometries of an affine homogeneous convex domain

By Tadashi TSUJI

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Introduction

Let Ω be a convex domain in the n-dimensional real number space $\boldsymbol{R}^{n}$, not containing any affine line, and let $G(\Omega)$ be the Lie group of all affine transformations on $\boldsymbol{R}^{n}$ leaving the domain Ω invariant. If the group $G(\Omega)$ acts transitively on Ω, then Ω is said to be *(affine) homogeneous*. By using the characteristic function φ of Ω, we can define a $G(\Omega)$-invariant Riemannian metric g_{Ω} on Ω as follows:

$$g_{\Omega}=\sum_{1\leqslant i,j\leqslant n}\frac{\partial^{2}\log\varphi}{\partial x^{i}\partial x^{j}}dx^{i}dx^{j},$$

where $(x^{1}, x^{2}, \cdots, x^{n})$ denotes a system of affine coordinates on $\boldsymbol{R}^{n}$. The Riemannian metric g_{Ω} is called the *canonical metric* of Ω (cf. [7], [8]). A homogeneous convex domain is said to be *reducible* if it is affinely equivalent to a direct product of homogeneous convex domains. A homogeneous convex domain is said to be *irreducible* if it is not reducible. We note that a homogeneous convex cone is a special case of a homogeneous convex domain.

For a homogeneous convex domain Ω, we denote by $I(\Omega)$ the group of all isometries of the homogeneous Riemannian manifold (Ω, g_{Ω}). Then, it has been proved that the groups $G(V)$ and $I(V)$ for an irreducible homogeneous convex cone V have the same connected component containing the identity element ([3], [6]).

The aim of the present paper is to extend the above result to homogeneous convex domains. Namely, we will prove the following statement: *If a homogeneous convex domain Ω is irreducible and not affinely equivalent to an elementary domain, then the groups $G(\Omega)$ and $I(\Omega)$ have the same connected component containing the identity element* (Theorem 6.1). The definition of an elementary domain will be given in § 3. In order to prove the above result, we will need the theory of T-algebras developed by Vinberg [8], [9], and also, we will make use of the results obtained in [5], [6] and [7].