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Foliations transverse to the turbulized foliations of punctured torus bundles over a circle

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§ 1. Introduction

Tamura and Sato [7] began the study of codimension-one foliations transverse to given codimension-one foliations. They regarded such foliations as structures of given foliated manifolds. At this viewpoint, the fundamental probelms are to determine whether such foliations exist or not and to classify them when they exist.

CONVENTION. In this paper, foliations are always transversely orientable, of codimension one and of class C^{∞}, unless stated otherwise.

In order to state the known results and ours, we introduce some notations. Let $\Sigma_g(h)$ be a compact manifold obtained from the closed surface Σ_g of genus g by deleting h small disjoint open 2-disks, where h is a positive integer. Take an orientation preserving C^{∞} diffeomorphism $\phi: \Sigma_g(h)\to\Sigma_g(h)$ and consider an equivalence relation $\sim$ on $\boldsymbol{R}\times\Sigma_g(h)$ determined by

$$(t, x)\sim(t', x') \qquad \text{if} \quad t'=t+1 \text{ and } x'=\phi(x),$$

where $t, t'\in\boldsymbol{R}$ and $x, x'\in\Sigma_g(h)$. Then the quotient space $E(\Sigma_g(h); \phi)=\boldsymbol{R}\times\Sigma_g(h)/\sim$ is a $\Sigma_g(h)$ bundle over $S^1=\boldsymbol{R}/\boldsymbol{Z}$ with the projection $\pi: E(\Sigma_g(h); \phi)\to S^1$ defined by $\pi([t, x])=[t]$ for $(t, x)\in\boldsymbol{R}\times\Sigma_g(h)$. We treat $\boldsymbol{R}$ and Σ_g as oriented manifolds. Hence $\Sigma_g(h)$ and $E(\Sigma_g(h); \phi)$ are consequently oriented. Take a continuous map $\sigma: \partial E(\Sigma_g(h); \phi)\to\{1, -1\}$. We have a foliation $\mathscr{F}(\Sigma_g(h); \phi)^{\sigma}$ of $E(\Sigma_g(h); \phi)$ by turbulizing the bundle foliation $\{\pi^{-1}(x)\}_{x\in S^1}$ as in Figure 1.1 (see Nishimori [4] for the precise definition).

We have $\Sigma_0(1)=S^2(1)=D^2$ and $E(D^2; \mathrm{id})=S^1\times D^2$. Note that $\mathscr{F}(D^2; \mathrm{id})^1$ (or $\mathscr{F}(D^2; \mathrm{id})^{-1}$) is a plus (or minus) Reeb component $\mathscr{F}_R^+$ (or $\mathscr{F}_R^-$) in Tamura-Sato [7]. For mainfold E with a diffeomorphism $f: E\to E(\Sigma_g(h); \phi)$, we denote by $\mathscr{F}(E)^{\sigma\circ f}$ the induced foliation $f^*\mathscr{F}(\Sigma_g(h); \phi)^{\sigma}$. Note that $\mathscr{F}(E)^{\sigma\circ f}$ is unique up to C^0 isomorphism.

The known results are as follows. Tamura and Sato [7] classify the foliations of $S^1\times D^2$ transverse to the Reeb component $\mathscr{F}_R^+$ $(=\mathscr{F}(S^2(1); \mathrm{id})^1)$ by introducing the notion of TS diagrams. Furthermore they proved that,