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On measures which are continuous by certain translation

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§ 1. Introduction.

Let G be a LCA group with the dual group $\hat{G}$. We denote by m_G the Haar measure on G. For $x\in G$, δ_x denotes the point mass at x. Let $M(G)$ and $L^1(G)$ be the measure algebra and the group algebra respectively. For a subset E of $\hat{G}$, $M_E(G)$ denotes the space of measures in $M(G)$ whose Fourier-Stieltjes transforms vanish off E. For a closed subgroup H of G, $H^{\perp}$ means the annihilator of H. Let μ be a measure in $M(G)$. Then, as well known, the fact that $\mu\in L^1(G)$ can be caracterized by

(1. 1) $$\lim_{x\to 0}\|\mu-\mu*\delta_x\|=0 .$$

Our first purpose in this note is to characterize the class of measures μ in $M(G)$ with the following property

(1. 2) $$\lim_{\substack{y\to 0\\ y\in H}}\|\mu-\mu*\delta_y\|=0 .$$

When there is a continuous homomorphism ϕ from the reals R into G, deLeeuw and Glicksberg proved in [2] that ϕ-analytic measures $\mu\in M(G)$ satisfy $\lim_{t\to 0}\|\mu-\mu*\delta_{\phi(t)}\|=0$. The second purpose in this paper is to give a theorem corresponding to theirs under our setting. As an extension of a theorem of Bochner, one of the authors proved in [8] that the product set of a Riesz set and a small p set is a small p set. As a corollary our second theorem, we shall prove that the product set of a small p set and a small q set is a small $\max(p, q)$ set. In section 2 we state our results, and we give their proofs in sections 3 and 4.

§ 2. Notations and Results.

DEFINITION 2. 1. *Let G be a LCA group and H a closed subgroup of G. A Borel set E in G is called a H-null set if $m_H(\{t\in H: t+x\in E\})=0$ for all $x\in G$.*

DEFINITION 2. 2. *For a positive integer p, a closed set E in $\hat{G}$ is called*